# Quality Control Through Game Theory of a Cascading Multi-Robot Machine Vision System

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Abstract—In this paper we analyze automated probabilistic quality control from a game theoretic point of view. Quality control is a key component of many industrial production lines and in the recent years there has been a push to automate this task, thanks to the advances in industrial manipulators with machine learning and vision capabilities. We formalize a serial multi-robot quality control model and analyze it in comparison with single-robot models, both theoretically and through some instance scenarios.

*Index Terms*—Networked manufacturing; Robotic assembly; Production control; Machine vision; Game theory.

#### I. INTRODUCTION

Quality control is a key step in most industrial production lines and processes. Checking that goods are up to the company's standards is traditionally a task for specialized human personnel, resisting automation due to the higher complexity and lower accuracy of automatic methods. However, these tasks are usually time consuming and repetitive, making them unappealing to the human operators [1]. Also, due to the time consuming nature of human quality control most goods are checked by random sampling [2]. Thus, the ever increasing demand for quality [3], [4] along with the development of new machine learning and deep learning techniques [5]–[7], has led to the rise of automated quality control.

Starting from these foundations, we explore a cascade of multiple classifiers to better trade-off individual classifier costs and overall performance using game theoretical methods. With respect to a single-controller system, cascading has the further objective of further performance improvements from just a better quality of each element [8], [9]; however, it poses several issues to network coordination [10].

This idea was inspired by boosting machine learning algorithms, in particular the approach of [11] to face recognition; yet, this and similar papers do not formally derive bounds on optimal strategies. Similarly, there are several previous works analyzing the cost/performance trade-off during training [12], [13], showing considerable interest about the subject; yet, we could not find any paper tackling such a problem analytically.

Such concepts will be explored focusing on the case of a cascade of collaborative robots employed in a quality control task, analyzing how Game Theory can be used to derive optimal cooperative strategies and their bounds [14]–[16].

The rest of this paper is structured as follows: first, in Section II we introduce the problem setup and describe the

environment and tasks involved. In Section III we develop a game theoretical multiplayer model for our problem and analyze different solutions. Section IV discusses instead the single player model for the same task, which is compared to the previous one in Section V. In Section VI, we conclude with final remarks and future directions.



Fig. 1. Reference scenario

# II. PROBLEM SETUP

Consider a 4.0 factory with automated quality control located at the end of a production line [8], [17]. The system, depicted in Fig. 1, consists of a conveyor belt carrying the goods exiting the production line, a series of N robots with machine vision systems, and a collecting bin at the end of the conveyor belt. The goods can be defective or quality products with probabilities d and 1-d, respectively. Each robot receives the result of the scanning of the object passing under its vision system and can decide whether to accept the good by picking it out of the conveyor belt, or discard the piece and let it travel further along the conveyor. The result of each scan is shared with the other robots, although those further along the belt cannot immediately act on a piece that has yet to reach them.

We consider the following cost/return parameters: (i) a fixed production cost for the good  $c_p$ ; (ii) a fixed gross return from the sale of quality goods  $g_p$ ; (iii) a fixed cost for refunding a faulty good, including e.g. the loss of reputation for the company  $c_d > g_p$ ; (iv) a parametric cost for operating a robot plus its corresponding vision system  $c_k$ , where  $k \in [1, ..., N]$ is the robot index. For ease of notation, we introduce  $c_r = c_d - g_p > 0$  as the net cost of selling a defective good.

Costs  $c_k$  are a function of the accuracy of the vision system employed. A suitable choice, assuming  $p_k$  is the probability of having a correct reading from machine vision system k, is:

$$c_k = \alpha \frac{p_k}{(1 - p_k)} \tag{1}$$

since the cost should increase slowly with low accuracy and sharply when the accuracy gets higher. This reflects the ease

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TABLE I LIST OF PARAMETERS AND NOTATIONS

term	meaning
d	defective products rate
$c_p$	per-item production cost
$g_p$	per-item sale price
$c_d$	per-item refund cost
$c_k, \ k \in \{1, 2\}$	cost of operating robot $k$
$c_r = c_d - g_p$	loss when selling a defective item
$p_k, \ k \in \{1, 2\}$	robot $k$ detection accuracy

of having a mediocre detection and the difficulty in increasing the accuracy above a certain threshold [18]. Parameter  $\alpha$  can be used to finetune the steepness of the function. This is just an example of what we might find in practice, but it is by no means the only option: we may model an operational cost, an initial investment divided over the expect lifetime of the line, or a mixture of both. Another possibility lies in exploiting  $c_k$  to represent the time required to perform inference: while this is often negligible, in some industrial setups the production rate can become critical. The latter case is equivalent to quantifying the inference cost of a cascade of boosted classifiers.

We further assume that  $p_k > 0.5$ , which is not limiting as the reverse case with  $p_k < 0.5$  can be brought back to this condition by simply inverting the signal.

The net earnings of the company (without accounting for the cost of operating the robots) are: (i) sale of a quality good:  $g_p - c_p$ ; (ii) sale of a faulty good:  $-c_p - c_r$ ; (iii) discarding a good (any type):  $-c_p$ . Thus, the expected gain for a single product when no quality check is employed is

$$G_0 = (1 - d) \cdot (g_p - c_p) + d \cdot (-c_p - c_r) = (1 - d) \cdot g_p - d \cdot c_r - c_p$$
(2)

meaning that the production cost is always incurred, there is a gross gain  $g_p$  when the product is up to standards (with probability 1 - d), and a gross loss  $c_r$  when the product is faulty (with probability d). The reader can refer to Table I for a list of the parameters introduced above.

Our focus is on whether it is profitable for the company to use one or more robotic quality inspectors and under which conditions. We employ game theory to formalize the mathematical constraints involved and the approaches to the best solution for a business strategy [19].

### III. GAME MODEL

The case with just one robot along the belt boils down to single player optimization. Its analysis is included in Section IV for completeness' sake and to compare it with the multirobot case. For now, consider N > 1 robots along the line, focusing on the N = 2 for simplicity. An extension to N > 2 would be conceptually immediate.

Robots  $R_1$  and  $R_2$  are assumed to receive correct quality scans from their associated vision systems with respective probabilities  $p_1$  and  $p_2$ , modeled as Nature's choices. The cost of operating them is  $c_1$  and  $c_2$  computed according to (1). We suppose  $p_2 > p_1$  so that the second, more powerful but costly, robot can correct the mistakes made by the first one, while not



Fig. 2. Complete two-player game in extensive form

affecting the costs as much as if both had the highest precision. Each player receives a signal from its vision system and can accordingly choose between two actions: to accept (A) the product and take it out of the line or discard (D) the product and let it go forward. All scan results are common knowledge, since the line is an engineered collaborative environment [10].

The quality of each product is modeled once again as Nature's decision, and the robots play sequentially. To get reasonable quantities as the robots' payoffs, we decided to distribute the company's revenues between the players, proportionally to their involvement. The production cost is not considered in the individual utilities since it is fixed and not depending on their choices. We need to factor it in again when computing the company's payoff. Thus, there are 5 end results (and corresponding payoffs, denoted by  $u_k$ ):

1)  $R_1$  accepts a quality product:  $u_1(1) = g_p - c_1, u_2(1) = 0$ 

- 2)  $R_1$  accepts a faulty product:  $u_1(2) = -c_r c_1, u_2(2) = 0$
- 3)  $R_2$  accepts a quality product, while  $R_1$  wrongly discarded it:  $u_1(3) = -c_1, u_2(3) = g_p c_2$

4)  $R_2$  accepts a faulty product, while  $R_1$  correctly discarded it:  $u_1(4) = -c_1$ ,  $u_2(4) = -c_r - c_2$ 

5) The product is discarded, after both robots checked it:  $u_1(5) = -c_1, u_2(5) = -c_2$ 

We denote with u(x) the tuple  $(u_1(x), u_2(x))$ . This way, we reflect each player's actual contribution, while keeping the company profit computation as the sum of the two individual utilities minus the (fixed) production cost [20]. This leads to the complete (but imperfect) information dynamic game depicted in extensive form in Fig. 2, where the game ends without actions from  $R_2$  in the cases where the first robot accepts, since the product is not on the line anymore.

Player  $R_1$  has 2 information sets corresponding to the uncertainty of whether the good is faulty or not.  $R_2$ , on the other hand, has 4 information sets, since it has the same uncertainty as  $R_1$ , plus it is also uncertain whether the signal of the first robot was correct or not. Because of all choices being binary, this leads to 4 and 16 strategies for  $R_1$  and  $R_2$ , respectively. Also due to its many parameters, the complexity becomes too high for an exhaustive analysis and must be reduced with an engineering perspective. E.g., we can limit  $R_2$ 's play to the best responses to all of  $R_1$ 's strategies. By backward induction, we can thus solve the game [21].

#### A. Analysis of $R_2$ 's strategies

We choose  $R_2$ 's strategy of always following the signal, due to these reasons. (i) Strategies of always accepting/discarding are pointless and could be achieved by just removing the robot. (ii) Strategies always blindly following  $R_1$  in at least one case are not useful, since the second robot is supposed to correct the first one's mistakes. This would restrict the player's capabilities, and not achieve cost-effective management. (iii) Given that the accuracy of each signal is higher than 0.5, we also discard strategies going against signals.

We want to impose that "follow the signal" for  $R_2$  is the only best response to player  $R_1$ 's moves as it strictly dominates the others. Let  $K_{X,Y}$  be the probability that the product is in state  $X \in \{A, D\}$  (quality or faulty), and the robot(s)  $Y \in \wp\{1, 2\}$  correctly detected such state. E.g.,  $K_A$  is the probability that no robot detects that the piece is up to standards, i.e.,  $(1-d)(1-p_1)(1-p_2)$ , while  $K_{D,2}$  is the probability that only  $R_2$  detects a faulty product, i.e.,  $d(1-p_1)p_2$ . Applying Bayes' rule to the 4 information sets of  $R_2$ , and imposing the preference of the appropriate action in each one of them, we can derive the following conditions on the parameters, one for each information set:

$$c_r \cdot K_{D,12} > g_p \cdot K_A \tag{3}$$

$$c_r \cdot K_{D,2} > g_p \cdot K_{A,1} \tag{4}$$

$$g_p \cdot K_{A,2} > c_r \cdot K_{D,1} \tag{5}$$

$$g_p \cdot K_{A,12} > c_r \cdot K_D \tag{6}$$

Assuming (4) and (5) hold, the following inequalities are satisfied, implying (3) and (6) as well if  $p_1, p_2 > 0.5$ :

$$c_r \cdot K_{D,12} > c_r \cdot K_{D,2} > g_p \cdot K_{A,1} > g_p \cdot K_A$$
  
$$g_p \cdot K_{A,12} > g_p \cdot K_{A,2} > c_r \cdot K_{D,1} > c_r \cdot K_D$$

So, the only constraints for which  $R_2$  is always better off following its signal are (4) and (5). The inequalities can be rewritten by highlighting  $g_p/c_r$ , obtaining

$$\frac{d}{1-d} \cdot \frac{p_1}{1-p_1} \cdot \frac{1-p_2}{p_2} < \frac{g_p}{c_r},$$

$$\frac{g_p}{c_r} < \frac{d}{1-d} \cdot \frac{1-p_1}{p_1} \cdot \frac{p_2}{1-p_2}$$
(7)

These constraints on  $g_p/c_r$  hold since if this ratio is too high, the incentive to sell each a product is too strong to follow a "discard" signal. On the other hand, if it is too low, then the risk of selling a faulty product is too high to follow any "accept" signal. This must be weighted on the prior probability of having faulty goods and the accuracy of the two robots.

# B. Analysis of $R_1$ 's strategies

With the above choice of  $R_2$ 's strategy,  $R_1$  can see the original game as equivalent to what reported in Table II, where rows and columns list  $R_1$ 's and  $R_2$ 's strategies, respectively. The action list on each strategy follows the tree in Fig. 2,

TABLE II CHOICES OF  $R_1$  when  $R_2$  follows the signal ("DADA")

$$\begin{array}{c|c} & \text{DADA} \\ & g_p \cdot (1-d) - c_r \cdot d - c_1, \\ & 0 \\ \text{AD} & \begin{array}{c} g_p \cdot (1-d)(1-p_1) - c_r \cdot d \cdot p_1 - c_1, \\ & (K_{D,2} + K_{A,1}) \cdot u_2(5) + K_D \cdot u_2(4) + K_{A,12} \cdot u_2(3) \\ & g_p \cdot (1-d) \cdot p_1 - c_r \cdot d \cdot (1-p_1) - c_1, \\ & (K_{D,12} + K_A) \cdot u_2(5) + K_{D,1} \cdot u_2(4) + K_{A,2} \cdot u_2(3) \\ \end{array} \\ \begin{array}{c} \text{DD} & \begin{array}{c} -c_1, \\ & -c_2 + g_p \cdot (K_{A,2} + K_{A,12}) - c_r \cdot (K_D + K_{D,1}) \end{array} \end{array}$$

from left to right, e.g., strategy "AD" for  $R_1$  means that "A" is played in the blue information set and "D" in the red one.

For pure strategies, only "DA" is desirable, since "AA" and "DD" would make the robot useless (but still costing money to run), while "AD" contradicts the choice of  $p_1 > 0.5$ . If we move to mixed strategies, the reasoning is not as simple.

Given that  $R_1$  knows that  $R_2$  is further along the line, and that it has greater classification abilities, it could make sense for  $R_1$  to discard some products even if its signal is positive, so as to let  $R_2$  make a better decision, thus generating interest in analyzing the mixed strategy with support  $\{DA, DD\}$ . This should be offset with the lack of reward for  $R_1$  for quality products correctly recognized, thus avoiding purely falling into strategy "DD".

1) Mixed Strategies: For  $R_1$  to be able to play a mixed strategy in this setup, it must be that all strategies in the support have the same utility  $(u_1)$ , which must be greater than the utilities of strategies outside it [22]. Since  $R_2$  does not need to be convinced to play its only strategy, the choice of m (which is the probability of  $R_1$  choosing "DA") is free in the range (0, 1).

Solving the equality condition, the result is:

$$g_p \cdot (1-d) \cdot p_1 = c_r \cdot d \cdot (1-p_1)$$

while the two inequality conditions imply:

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$$g_p \cdot (1-d) < c_r \cdot d$$
$$g_p \cdot (1-d)(1-p_1) < c_r \cdot d \cdot p_1$$

All these restrictions are compatible with the conditions on the strategy of  $R_2$ , as long as the parameters are chosen correctly. So, at least from a mathematical point of view, there are practical instances that allow for this mixed equilibrium. This does not mean that it is profitable from the company's point of view; in fact, if we let  $u_{2,DA}$  be the payoff for  $R_2$  whenever  $R_1$  plays DA, and  $u_{2,DD}$  the payoff for  $R_2$  when  $R_1$  plays DD, then the company utility is

$$u_1 + u_2 - c_p = u_1 + m \cdot (u_{2,DA}) + (1 - m)u_{2,DD} - c_p$$
 (8)

If  $u_{2,DA} > u_{2,DD}$  then it is most convenient to set m to 1, returning to a pure strategy; in the symmetrical case it is best to set m - 1 to 1. The only option where we could reasonably have a true mixed equilibrium with any m is if the two quantities are equal. If this holds, then the company's payoff in the mixed equilibrium is exactly the same as if the



Fig. 3. In the orange and purple regions, (12) or (7) is the most stringent condition, respectively.

first robot always plays "DD". Seeing this, the company can just remove the first player and get a greater payoff since  $c_1$ would not be paid anymore. We hence did not go into greater detail about the mixed strategy calculations, since it is always outperformed by pure strategies or single player scenarios.

2) Pure Strategies: Thus, the only Nash Equilibrium in pure strategies is "DA" for player  $R_1$  and "DADA" for  $R_2$ . After finding the conditions for  $R_2$ , we now focus on those for  $R_1$ , which are ( $\succ$  stands for "strictly dominates")

"DA" 
$$\succ$$
 "DD" if:  $g_p(1-d) > c_r \cdot d \cdot \frac{1-p_1}{p_1}$  (9)

"DA" 
$$\succ$$
 "AD" if:  $(g_p(1-d) + c_r \cdot d) \cdot (2p_1 - 1) > 0$  (10)

"DA" 
$$\succ$$
 "AA" if:  $g_p(1-d) < c_r \cdot d \cdot \frac{p_1}{1-p_1}$  (11)

We can drop (10) since the first factor is a sum of positive quantities and the second factor is always positive due to our assumption  $p_1 > 0.5$ . The remaining inequalities impose the following range on  $g_p/c_r$ , with the same intuitions as (7):

$$\frac{d}{1-d} \cdot \frac{1-p_1}{p_1} < \frac{g_p}{c_r} < \frac{d}{1-d} \frac{p_1}{1-p_1}$$
(12)

Merging this constraint with (7) leads to two ranges, i.e., (7) if  $p_1 > (1 + \sqrt{1 - p_2/p_2})^{-1}$ , and (12), otherwise.

The meaning of these conditions is that if  $p_1$  is close enough to  $p_2$ , then the second player has doubts when it wants to contradict player 1's choices and so it has the strictest conditions. Otherwise, the second player can just follow its signal without second thoughts and, since  $p_2 > p_1$ , it is the first player that must check that its detector quality is sufficient. The domains of the two ranges are shown in Fig. 3.

To end this analysis, we compute the company's payoff for the resulting Nash Equilibrium:

$$G_2 = g_p \cdot K_{A,U} - c_r \cdot K_{D,\bar{I}} - c_p - c_1 - K_2 \cdot c_2 \qquad (13)$$

where  $K_{A,U} = (1-d) \cdot (p_1+p_2-p_1p_2)$  is the probability that a product is of good quality and gets correctly labeled by either player;  $K_{D,\bar{I}} = d \cdot (1-p_1p_2)$  is the probability that a product is faulty and still passes both checks;  $K_2 = d p_1 + (1-d)(1-p_1)$ is the probability that the second robot gets to play a move.

# IV. SINGLE PLAYER SETUP

This setup is solvable by single agent optimization, without any game theory. We still include this analysis to make a comparison with the two-player model. The payoffs are computed as in the previous cases, only, there is no option of letting the second robot play. Following the same reasoning previously made, we choose "D" in the blue information set and "A" in the red information set. Hence,

$$\frac{d}{1-d} \cdot \frac{1-p}{p} < \frac{g_p}{c_r} < \frac{d}{1-d} \cdot \frac{p}{1-p} \tag{14}$$

If we replace p with  $p_1$ , the inequality expresses the same constraints as (12) since the payoff of  $R_1$  in that case is that of a single player. The company's payoff in this setup is:

$$G_1 = -c_s - c_p - c_r \cdot d \cdot (1-p) + g_p \cdot (1-d) \cdot p \quad (15)$$

where  $c_s$  is the cost of operating the robot, modeled by (1). If we compute  $dG_1/dp$  and set it equal to zero we obtain (as the only feasible solution)

$$p^* = 1 - \sqrt{\frac{\alpha}{g_p \cdot (1-d) + c_r \cdot d}}$$

which is a maximum. Should robot costs change, the value of  $p^*$  is to be recomputed accordingly.

#### V. MODEL COMPARISONS

# A. Closed form analysis

The simplest case to consider is quality control being done more conveniently with just one robot instead of none:

$$G_1 > G_0 \Leftrightarrow g_p \cdot (1-d) \cdot (1-p) + c_s < c_r \cdot d \cdot p$$
 (16)

which is equivalent to

$$\frac{g_p}{c_r} < \frac{d}{1-d} \cdot \frac{p}{1-p} - \frac{c_s}{c_r \cdot (1-d) \cdot (1-p)}$$
(17)
  
upper range limit

This constraint is identical to "DA dominates AA" but is made stricter by the fact that the robot's cost does not cancel out anymore. The physical meaning is that if the gain is proportionally much bigger than the return cost then it makes more sense to accept the full price for the refunds and skip quality control altogether. Thus, if we satisfy the inequality above we are automatically below the upper limit of the range for operating the robot in the desired fashion.

Analytically comparing the two-robot setup with the other two models is instead quite complex to do in closed form and it does not give any more intuitive insight than what already presented. For example, when imposing  $G_2 > G_0$  we get:

$$\frac{q_p}{c_r} < \frac{d - K_{D,\bar{I}}}{(1-d) - K_{A,U}} - \frac{c_1 + K_2 \cdot c_2}{(1-d) - K_{A,U}}$$
(18)

with a similar meaning to (17).

Furthermore, it proved too complex to solve for  $p_1^*, p_2^*$  in closed form with our choice of robot cost function, even when employing symbolic solvers. It is instead easier to evaluate the different models with numeric methods when fixing the values of  $g_p, c_p, c_r, d$  and  $\alpha$ , as we do in the next section.

TABLE III SAMPLE SCENARIOS, CHECKED FOR THE CONSTRAINTS ON  $g_p/c_r$ 

Scenario 1	G2 is best, both G1 and G2 constraints satisfied
parameters	$g_p = 6.6, c_p = 2.2, c_r = 16.5, d = 0.2, \alpha = 0.2$
$G_0$	-0.21
$G_{1}^{*}$	0.66 at $p = 0.847$
$G_2^*$	0.72 at $p_1 = 0.785, p_2 = 0.867$
$g_p/c_r$ constraints	$= 0.4;  (G_1): \in [0.05, 1.39];  (G_2): \in [0.14, 0.45];$
Scenario 2	G2 is best, only G1 constraint satisfied
parameters	$g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.02$
$G_0$	2.42
$G_1^*$	2.99 at $p = 0.947$
$G_2^*$	3.27 at $p_1 = 0.897, p_2 = 0.955$
$g_p/c_r$ constraints	$= 0.5; (G_1): \in [0.01, 2.01]; (G_2): \in [0.05, 0.27];$
Scenario 3	C0 is best both C1 and C2 constraints violated
Sechario 5	do is best, both di and d2 constraints violated
parameters	$g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2$
parameters $G_0$	$g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2$ 3.41
parameters $G_0$ $G_1^*$	$g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2$ 3.41 1.92 at $p = 0.830$
parameters $G_0$ $G_1^*$ $G_2^*$	$g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2$ 3.41 1.92 at $p = 0.830$ 2.70 at $p_1 = 0.717, p_2 = 0.835$
parameters $G_0$ $G_1^*$ $G_2^*$ $g_p/c_r$ constraints	$\begin{array}{l} g_p = 6.6,  c_p = 2.2,  c_r = 13.2,  d = 0.1,  \alpha = 0.2 \\ 3.41 \\ 1.92 \ \text{at} \ p = 0.830 \\ 2.70 \ \text{at} \ p_1 = 0.717,  p_2 = 0.835 \\ = 0.5;  (G_1): \in [0.01, 0.26];  (G_2): \in [0.03, 0.11]; \end{array}$
parameters $G_0$ $G_1^*$ $G_2^*$ $g_p/c_r$ constraints Scenario 4	$\begin{array}{l} \text{Go is best, bold G1 and G2 constraints violated} \\ g_p = 6.6,  c_p = 2.2,  c_r = 13.2,  d = 0.1,  \alpha = 0.2 \\ 3.41 \\ 1.92 \text{ at } p = 0.830 \\ 2.70 \text{ at } p_1 = 0.717,  p_2 = 0.835 \\ = 0.5;  (G_1): \in [0.01, 0.26];  (G_2): \in [0.03, 0.11]; \\ \hline \text{G1 is best, both G1 and G2 constraints satisfied} \end{array}$
parameters $G_0$ $G_1^*$ $G_2^*$ $g_p/c_r$ constraints Scenario 4 parameters	$\begin{array}{l} \text{G0 is best, bold G1 and G2 constraints violated} \\ g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2 \\ 3.41 \\ 1.92 \text{ at } p = 0.830 \\ 2.70 \text{ at } p_1 = 0.717, p_2 = 0.835 \\ = 0.5;  (G_1): \in [0.01, 0.26];  (G_2): \in [0.03, 0.11]; \\ \hline \text{G1 is best, both G1 and G2 constraints satisfied} \\ g_p = 6.6, c_p = 2.2, c_r = 16.5, d = 0.3, \alpha = 0.1 \\ \end{array}$
parameters $G_0$ $G_1^*$ $G_2^*$ $g_p/c_r$ constraints Scenario 4 parameters $G_0$	$\begin{array}{l} \text{G0 is best, bold G1 and G2 constraints violated} \\ g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2 \\ 3.41 \\ 1.92 \text{ at } p = 0.830 \\ 2.70 \text{ at } p_1 = 0.717, p_2 = 0.835 \\ = 0.5;  (G_1): \in [0.01, 0.26];  (G_2): \in [0.03, 0.11]; \\ \hline \text{G1 is best, both G1 and G2 constraints satisfied} \\ g_p = 6.6, c_p = 2.2, c_r = 16.5, d = 0.3, \alpha = 0.1 \\ -2.53 \end{array}$
parameters $G_0$ $G_1^*$ $G_2^*$ $g_p/c_r$ constraints Scenario 4 parameters $G_0$ $G_1^*$	$\begin{array}{l} \text{G0 is best, bold G1 and G2 constraints violated} \\ g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2 \\ 3.41 \\ 1.92 \text{ at } p = 0.830 \\ 2.70 \text{ at } p_1 = 0.717, p_2 = 0.835 \\ = 0.5;  (G_1): \in [0.01, 0.26];  (G_2): \in [0.03, 0.11]; \\ \hline \text{G1 is best, both G1 and G2 constraints satisfied} \\ g_p = 6.6, c_p = 2.2, c_r = 16.5, d = 0.3, \alpha = 0.1 \\ -2.53 \\ 1.09 \text{ at } p = 0.928 \end{array}$
parameters $G_0$ $G_1^*$ $g_2/c_r$ constraints Scenario 4 parameters $G_0$ $G_1^*$ $G_2^*$ $G_2$	$\begin{array}{l} \text{Go is best, bold G1 and G2 constraints violated} \\ g_p = 6.6, c_p = 2.2, c_r = 13.2, d = 0.1, \alpha = 0.2 \\ 3.41 \\ 1.92 \text{ at } p = 0.830 \\ 2.70 \text{ at } p_1 = 0.717, p_2 = 0.835 \\ = 0.5; \ (G_1): \in [0.01, 0.26]; \ (G_2): \in [0.03, 0.11]; \\ \text{G1 is best, both G1 and G2 constraints satisfied} \\ g_p = 6.6, c_p = 2.2, c_r = 16.5, d = 0.3, \alpha = 0.1 \\ -2.53 \\ 1.09 \text{ at } p_1 = 0.928 \\ 0.91 \text{ at } p_1 = 0.902, p_2 = 0.941 \end{array}$



Fig. 5. Scenario 1:  $G_2$  versus  $p_1$  and  $p_2$ 

### B. Case studies

We discuss the scenarios listed in Table III, drawing interesting conclusions. While the parameters are theoretic figures, they can be matched with real industrial contexts [8], [23].

Scenario 1 (see Figs. 4–5) has all models within bounds, with the two-robot setup performing the best, proving that using two robots may be convenient. Working without any robot would lead to a net loss, justifying the need for quality



Fig. 7. Scenario 2:  $G_2$  versus  $p_1$  and  $p_2$ 

control. Also in Scenario 2 (see Figs. 6–7) the two-robot setup outperforms the others; but, upon further inspection, such a setup is operating outside the NE constraints. In this case, we should re-analyze the game to find out the resulting equilibrium, and possibly modify the setup accordingly. Finally, Scenarios 3 and 4, are counterexamples, where the best models are without quality control and with a single robot, and are reported in Figs. 8–9 and Figs. 10–11, respectively. All of this showcases the operational conditions can affect the outcomes.

#### VI. CONCLUSIONS

We employed game theory to merge predictors considering the cost of accuracy for robotic production lines. We compared models with no predictor, one predictor, and two cascaded predictors following their signals. The results are not confined to quality control, nor an industrial setup: similar approaches can be applied to other tasks [24], [25]. For each model, nontrivial constraints were obtained, as well as payoff optimization techniques; we also showed an array of sample scenarios.

Our game theoretic analysis of accuracy vs. prediction time/cost at inference time with a multi-classifier approach may serve as a foundation for other models, by relaxing some assumptions. E.g., in the two-classifier model, probabilities  $p_1$ and  $p_2$  can be correlated [26], or the false-positive and falsenegative probabilities be different [27], [28]. A cost relation other than (1) can be used, e.g., if we represent the time spent on inference, we may model  $c_2$  to account for the second robot receiving fewer products, impacting less on production time. Finally, mixed strategy setups can be explored.



Fig. 9. Scenario 3:  $G_2$  versus  $p_1$  and  $p_2$ 

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Fig. 11. Scenario 4:  $G_2$  versus  $p_1$  and  $p_2$ 

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