Harsanyi's Equilibrium Selection for Distributed Sources Minimizing Age of Information

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***Abstract*—Efforts to minimize Age of Information (AoI) in communication networks, particularly within energy-constrained devices in the Internet of Things (IoT), have prompted extensive research into resource management techniques. This study explores the optimization of AoI over a finite horizon in the context of distributed IoT environments. We first frame the scenario of N distributed equivalent sources as a multi-agent coordination game, then we address the inefficiency of the resulting equilibria, quantified through the Price of Anarchy. We find the latter to be significant (higher than 1.5) already for few sources, and increasing in the number of players. Leveraging Harsanyi's theoretical framework for equilibrium selection, we argue for the importance of preplay communication for AoI efficiency, and suggest how this can be implemented in the IoT without resorting to full centralization.**

***Index Terms*—Age of Information; Internet of Things; Data acquisition; Game theory; Equilibrium selection.**

I. Introduction

The advancement of communication technologies has led us into the era of the Internet of Things (IoT), characterized by ubiquitous connectivity among a vast array of devices [1]. Despite its progress, challenges persist for efficient resource management in IoT networks, especially those constrained by energy limitations. One of such issues that is expected to become pressing in the future, in light of the increase in real-time services, is the provision of timely information to make accurate decisions on the system control [2].

To quantify freshness of information from a mathematical standpoint, a relatively recent line of research adopts Age of Information (AoI) as the metric of choice [3], seen as most important than average delay or throughput in real-time applications. Whenever a transmitter and receiver exchange status updates, AoI at the receiver is defined as [4]

$$\Delta(t) = t - \sigma(t) \tag{1}$$

where $\sigma(t)$ is instant of generation of the last correctly received transmission.

AoI minimization in communication networks corresponds to enhance the freshness of information delivery and is commonly subject to two requirements imposed by IoT devices. The first relates to energy and data link restrictions, which may result in the prohibition of persistent transmission, requiring the nodes to limit their activity [5], [6]. Another common limitation is that the management of multiple nodes happens without coordination, in a distributed fashion, due to scalability requirements on large-scale IoT scenarios [7].

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These challenges have led to a diverse array of techniques and methodologies aimed at mitigating AoI and optimizing resource utilization in constrained device networks. One notable avenue of research focuses on minimizing AoI over finite horizons [8], recognizing the critical importance of preserving resources while maintaining information freshness. Additionally, researchers have leveraged multiple mathematical methodologies, from queueing theory to constrained optimization, toward analyzing AoI dynamics [9], exploring optimal update policies to balance information update frequency and energy consumption effectively.

However, a line of relatively more recent papers deals instead with the application of *game theory* as a tool for optimizing resource allocation in various network scenarios. This can be seen as a way to combine distributed management of multiple agents toward the common goal of network optimization. In particular, in this paper we address the problem of AoI minimization with multiple equivalent distributed sources as an *anti-coordination game* [10]. However, we are aware that conventional game models face challenges in scaling to the complexity of IoT systems, which commonly results in poor performance, as will be confirmed by our preliminary results where we quantify the Price of Anarchy (PoA) of our system that turns out to be excessive and not scaling [11]. This asks for the development of non-conventional game-theoretic models tailored to address the unique characteristics of large-scale IoT deployments.

For this reason, in this paper we extend the analysis to include Harsanyi's concept of *equilibrium selection* [12], which may offer a better solution to the issue of multiple strategic saddlepoints in games giving inefficient results. This framework provides a way to choose a satisfactory outcome based on players' beliefs and the likelihood of different outcomes [13]. Integration of Harsanyi's theory alongside game-theoretic approaches may be the solution to coordination challenges and optimizing resource allocation in IoT environments, emphasizing the significance of preplay communication in achieving more desirable outcomes, particularly correlated equilibria. We seek to provide insights into effective resource management strategies for optimizing AoI performance and fostering coordination within IoT networks.

II. Related Work

The analysis of AoI in communication networks has garnered significant attention in recent years, particularly in the context of energy-constrained device networks. Researchers have explored various techniques to enhance the freshness of information delivery, addressing the challenges posed by constrained devices [4], [6], [8], [9], [14].

One notable line of research involves the minimization of AoI over a finite horizon in the presence of energy-constrained devices [8]. Considering that such devices rely on limited batteries, optimizing AoI performance becomes crucial to prolonging network lifetime while maintaining information freshness. In addition, researchers have explored the application of queueing theory to analyze AoI dynamics in constrained device networks [4], [6]. Paper [4] utilized queueing models to investigate optimal update policies for energy-constrained devices, aiming to strike a balance between information update frequency and energy consumption. Similarly, [6] examined the impact of different battery horizons on AoI performance, providing insights into optimal update strategies under energy constraints.

Furthermore, [14] considered AoI minimization over infinite horizons in IoT scenarios, emphasizing the importance of long-term freshness maintenance in energy-constrained networks. Their work contributes to understanding AoI dynamics beyond finite timeframes, offering insights for designing efficient communication protocols in IoT environments. Overall, all these studies underscore the importance of tailored approaches for AoI minimization in constrained device networks, considering factors such as energy efficiency, battery constraints, and long-term freshness maintenance.

Game theory is also used to optimize resources in network scenarios [9], as it offers a framework for analyzing distributed and interactive decision-making processes, but conventional models struggle with the scale of IoT systems. In response, non-conventional game theoretic models such as evolutionary games, mean field games, minority games, mean-field bandit games, and mean field auctions have emerged [7]. Game-theoretic approaches in relay-assisted channels have also shown how to improve efficiency [15].

III. Analysis

A. System model

We examine a multi-agent scenario where multiple IoT nodes send information to the same end point. The primary objective of these agents, herein referred to as information *sources*, is to optimize data freshness within the network environment. We consider an optimal update schedule over a finite horizon [8], during which we assume that each source can only send one update (otherwise, the same approach can be repeated over multiple fractions). For numerical convenience, the finite horizon is taken as normalized to $[0, 1]$.

We consider the standard AoI expression $\Delta(t)$ as per (1). This implies that upon each transmission, AoI is reset to zero. Our focus is on quantifying the average AoI within the time window, i.e.

$$\Delta := \int_0^1 A(t)dt. \tag{2}$$

An example of the evolution of $\Delta(t)$ is reported in Fig. 1, where 3 sources are considered and their transmission instants are set as τ_j, $j = 1, 2, 3$. Thus, it is easy to derive through geometric arguments that (2) leads to Δ being a function of the choices of τ_j, or equivalently, of the $N+1$ inter-transmission intervals $y_j = \tau_j - \tau_{j-1}$, for $j = 1, \dots, N+1$ with $\tau_0 = 0$ and $\tau_{N+1} = 1$, which leads to

$$\Delta(y) = \frac{1}{N} \sum_{i=0}^{N} \left[\frac{{y_i}^2}{2} \right] \tag{3}$$

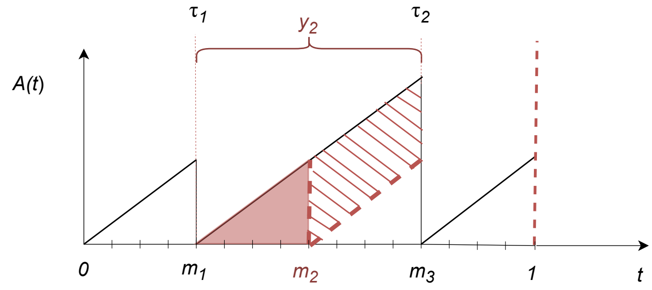

Fig. 1. Example timeline for the AoI evolution. Updates are planned at regular intervals by 3 sources, yet the second opportunity of transmission is skipped and the third is unnecessarily used by two sources. Consequently, the AoI experiences an increase as per the dashed surface.

If sources are fully coordinated, $\tau_j = j/(N+1)$ are the best points. Thereafter, we denote these values as *milestones* m_j.

Now, if the sources are instead independently managed by rational agents that do not communicate with one another, we can make the assumption that they still choose to transmit at one of the milestones. However, since this choice is totally uncoordinated, it can happen that a milestone is skipped whereas another has multiple (redundant) transmissions. This leads to an increase in the average AoI that is also shown in Fig. 1, as the shaded orange area. This is actually not the consequence of a transmission failure as in [8], or a collision as in [16], but rather that multiple uncoordinated sources choose the *same* milestone to send their information, thereby resulting in an inefficiency because of the redundant transmission. This, in turn, reflects on fewer transmission opportunities being exploited, which has the same impact on AoI as an erasure [9].

B. Game theoretic analysis

To formalize this scenario, we consider multiple sources as *players* in a static game of complete information, which can be represented as $\mathcal{G} = \mathcal{N}, \mathcal{A}, \mathcal{U}$, where $\mathcal{N}$ is the set of players, i.e., N independent sources of information. The action set $\mathcal{A} = (p_j)_j$, with $1 \leqslant j \leqslant N$, and $0 \leqslant p_j \leqslant 1$ denotes the set of possible actions of players, as the probability of transmitting in the jth milestone. The last component, $\mathcal{U}$ is a set representing the payoffs of the games with different numbers of players i and depends on the actions chosen by the players as (2).

This kind of situation is already studied in game theory as an *anti-coordination* game [10], since the strategic players are incentivized to choose different slots, but this is equivalent, under proper swap of their choices, to the pursuit of distributed coordination among multiple independent players. Yet, since the players are in reality *not* coordinated, they will possibly choose inefficiently. A naive solution to their choice would be to set all milestones as being chosen with equal probability, that is, $p_j = 1/N$ for all js. Yet, this is not an equilibrium point: the structure of the problem implies that some milestones can be preferred (particularly the ones in the middle), so we need to adjust the values of p_j at the equilibrium. This can be seen as a mixed strategy NE that necessitates the application of the *indifference theorem*, whereby players' decision probabilities are adjusted to render opponents indifferent among their available actions.

Given that our model encompasses a finite set of actions, the pursuit of a Nash equilibrium in mixed strategies demands adjustments to the action set $\mathcal{A}$. However, the computation of the theorem for N players scales pretty badly as it results in an $(N-1)$-th degree polynomial with terms such as $(p_1+p_2+\cdots+p_N)^{(N-1)}$. The computation can be improved by leveraging the game's symmetry, as players can be predicted to share identical probabilities for selecting the j-th or the $(N-j+1)$-th milestones, effectively halving the size of the action set. Additionally, we can impose a monotonicity constraint, i.e., $p_j < p_k$ whenever $j < k \leq \lceil N/2 \rceil$ that further simplifies the derivation. This results in Algorithm 1 to find the mixed strategy NEs.

Algorithm 1 Implementation of the Indifference theorem

Require: N probability vectors $\mathbf{p}$ with length N
$temp_{dev} = 100$ ▷ Initialize a temporary deviation variable with a high value
Ensure:
$\sum_{j=1}^{N} p_j = 1$; $p_j \geq 0$
$p_j = p_{N-j+1}$ ▷ Ensure symmetry
$p_j \leq p_{j+1} \leq ... \leq p_{N/2}, \text{where} 0 \leq j \leq N$ ▷ constraints that subsequent slots probabilities must be greater than previous
while Loop over length N of probability vector $\mathbf{p}$ **do**
  Calculate the polynomial coefficients
  Calculate expected values for different outcomes
  $u_j(1, coef) = u_j(2, coef) = ... = u_j(N/2, coef)$
  Calculate the average value of expected payoffs
  $dev = abs(u_1(1, coef) - average)$
  if $temp_{dev} \geq dev$ **then**
    $temp_{dev} = dev$
    $results = [u_1(1, coef), p]$
  end if
end while

Overall, the full set of NEs can be seen as comprising those in pure strategies, which imply full coordination, and mixed strategies, achievable through random distributed choices. Yet, the average AoI of the mixed strategy NE is close to that obtained when utilizing uniformly distributed probabilities for transmission at each milestone. Conversely, any of the pure strategy NEs, which correspond to all the players being perfectly anti-coordinated, is much more efficient.

Fig. 2 demonstrates the resulting average AoI in scenarios where players are perfectly coordinated and coordinated with probabilities corresponding to the mixed equilibrium across varying numbers of players. Equilibria in the mixed strategy are not the persistent ones [17] and in our case, it yields the unfavorable value of average information freshness. If we design the system to enable *preplay* communication then we will achieve a more attractive solution with a probability mixture of pure Nash equilibria [12], solution defined as *correlated equilibrium* which will result in a lower AoI.

To better see the inefficiency between perfect coordination and distributed choices, we plot the PoA, a metric used to quantify the inefficiency stemming from selfish behaviors [11], [13]. In our study, the PoA is simply the ratio of AoI in the mixed strategy NE (the worst one) vs any pure strategy equilibrium with round-robin-like choices, which is also the optimum. The AoI ratio of the uniform choices is plotted for comparison. As depicted in Fig. 3, the PoA starts already close to 1.5 for three players, and rapidly increases in N.

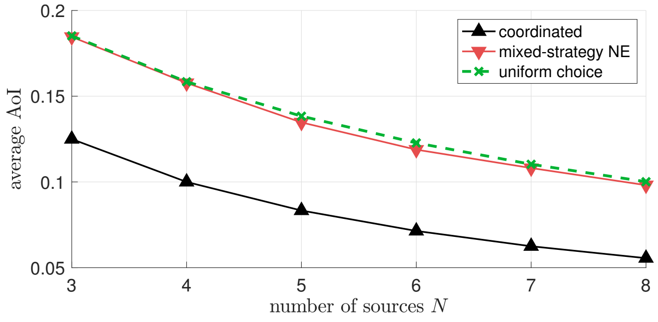

Fig. 2. Average value of AoI

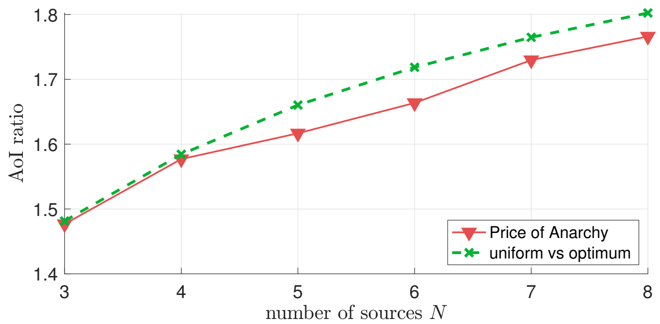

Fig. 3. Price of Anarchy

The observed trend underscores the importance of fostering cooperation and coordination among network participants to mitigate the detrimental effects of selfish behavior. Integrating *preplay communication* mechanisms [17] alongside the incorporation of tie-breakers holds promise for fostering outcomes more aligned with optimal coordination within systems. This integration stands to mitigate the Price of Anarchy by constraining the scope for suboptimal strategies.

IV. Harsanyi's Equilibrium Selection

Harsanyi's theory of equilibrium selection addresses the issue of how players in a game with complete information can select among multiple NEs to coordinate on a particular outcome [12]. This involves a further strategic layer over the traditional approach to non-cooperative games, where all the NEs are treated as equivalent, and implies instead further reasoning capabilities that are, however, not alien to devices empowered by machine learning (ML) techniques [18].

Instead of focusing on bilateral risk comparisons between pairs of equilibria, Harsanyi proposes the concept of *multilateral risk dominance*, which lies in determining the equilibrium that minimizes risk across all alternatives with relevant properties. By comparing the equilibria, players can choose the one that offers the lowest strategic risk. In this context, strategic risk pertains to the likelihood that a player's chosen strategy will not yield the best possible outcome given the strategies chosen by other players. While it may be impossible for players to entirely eliminate strategic risk, it can be significantly mitigated by selecting equilibria with the highest theoretical probability of realization [11]. In other words, players seek strategies that are robust and resilient against deviations by other players, thereby increasing the likelihood of achieving favorable outcomes. The equilibrium that emerges as the least risky choice, based on this evaluation, is considered the solution of the game.

Harsanyi suggests that the players directly evaluate the relative strength of different strategies in achieving favorable outcomes across the entire game, rather than solely within specific subgames. This involves the modification of the equilibrium selection criterion from a combination of payoff and risk dominance to solely relying on risk dominance. This adjustment is made in response to the nature of non-cooperative games and aligns with Aumann's theory [17], which emphasizes the limitations of achieving payoff dominant equilibria in such contexts. Note that in non-cooperative games, players act independently and pursue their own interests without coordinated agreements or communication. Even with the possibility of *preplay* communication, players cannot reliably enforce agreements to achieve payoff dominant equilibria. This is because there is no mechanism to ensure compliance with agreements, and players may have incentives to deviate from agreed-upon strategies to maximize their individual gains.

A final adjustment involves incorporating tie-breakers to address disparities between different equilibria. Prior theory [19] suggests that, in symmetric games, a symmetric equilibrium ought to be selected, but this often leads to suboptimal outcomes, with low stability and poor payoffs. In his updated theory [12], Harsanyi argues that, for scenarios where *preplay communication* is permitted, the optimal solution should manifest as a *correlated equilibrium*.

This reasoning is usually illustrated through an (anti) coordination game, where the traditional solution leans towards a mixed strategy equilibrium, primarily due to its symmetry. Harsanyi's theory suggests instead a correlated equilibrium in pure strategies, which offers a better long-term performance. This approach is also frequent in networking problems, but commonly interpreted as the result of a Stackelberg formulation [15].

Our AoI minimization problem presents a similar structure of an anti-coordination game [10], where the correlated equilibrium with preplay communication would correspond to a centralized solution, leading to a round-robin with pre-established role (or equivalently, a polling with a central arbitrator) [8]. Conversely, a distributed symmetric NE is an inefficient operating point as shown in the previous results.

We argue that a selection towards correlated equilibrium can be obtained in practical contexts through some external means, without resorting to a centralized management. One first option would be to break symmetry by means of a personalized preference toward a certain transmission milestone by each source. In many kinds of sensor networks, this would be easy to implement by leveraging individual characteristics of the nodes, e.g., the battery level, and would reflect in a Bayesian type from a game theoretic standpoint [20].

Alternatively, pushing the choice towards a correlated equilibrium can be regarded as a natural consequence of reinforcement learning that is an already consolidated technique in many multi-source AoI optimization problems [18].

V. Conclusions and Future Work

Our study explored AoI-optimal finite-horizon scheduling within networks comprising N agents [8]. We focused on the differences between perfect coordination, a scenario unlikely in practice, and distributed choices following a mixed strategy NE. To quantify the inefficiency resulting from lack of coordination, we computed the PoA [13].

As a prospective avenue for future investigation, we propose to integrate Harsanyi's theory for equilibrium selection [12] within multi-agent AoI minimization over a finite horizon. While the solution in the standard game theoretic form can be acceptable, it would lead to a mixed NE that was shown to be inefficient. Leveraging preplay communication may attain more favorable outcomes of the game, particularly correlated equilibria, which offer better performance for resource-constrained IoT systems.

We suggest that the main direction for future investigations ought to lie in the implementation of something equivalent to preplay communication, without explicitly resorting to centralized control, but rather leveraging particular aspects of the communication protocols (especially, carrier sense can be regarded in this way, as a way to garner information about the other users without explicit communication [21]) and reasoning capabilities in ML-empowered devices [18].

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