

A Benders Decomposition Approach to the Time Window Assignment Traveling Salesperson Problem with Stochastic Travel Times

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We study the Time Window Assignment Traveling Salesperson Problem with Stochastic Travel Times, a two-stage stochastic problem where the first-stage decisions involve the routing aspects and the customer time window definition. Second-stage decisions follow, which integrate real-world uncertainties, notably stochastic travel times, into the optimization process. The objective is to minimize the combined routing and time window cost, including penalties for earliness and lateness, marking a shift from a cost-focused routing strategy to a more balanced approach that considers both cost and service quality aspects in delivery operations.

We introduce a novel formulation inspired by a 3-index formulation for the Time-Dependent Traveling Salesperson Problem, and we report an extensive computational comparison of alternative models and solution methods from the literature. Additionally, we provide a set of benchmark instances characterized by two opposite scenario types, intended to facilitate future research. Our results show that the (by far) most effective solution method is an ad-hoc Benders Decomposition algorithm that leverages our new model, demonstrating substantial improvements over prior state-of-the-art exact solution methods.

Keywords: Travelling Salesman, Benders Decomposition, Stochastic Programming

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