```
  1  /***      A. LANGUASCO & A. ZACCAGNINI   ***/
  2  /***      Implementation of Pintz-Ruzsa's method paper   ***/
  3  /***      (as described in their paper paper on Acta Arith. 109,(2003))   ***/
  4
  5  -----------------------------------------------------
  6  -----------------------------------------------------
  7  Results PRmethod-KB:
  8  -----------------------------------------------------
  9  4/5 on a quad core pc
 10  -----------------------------------------------------
 11
 12  [languasc@labsrv0 ~]$ nice /usr/local/Gruppi/PariGP/bin/gp2c-run -pmy_ -g -W PRmethod-KB.gp
 13  PRmethod-KB.gp.c: In function `my_PintzRuzsa':
 14  PRmethod-KB.gp.c:107: warning: unused variable `my_j'
 15  PRmethod-KB.gp.c:107: warning: unused variable `my_l'
 16  PRmethod-KB.gp.c:107: warning: unused variable `my_i'
 17  PRmethod-KB.gp.c:107: warning: unused variable `my_i1'
 18  PRmethod-KB.gp.c:107: warning: unused variable `my_i2'
 19                   GP/PARI CALCULATOR Version 2.3.2 (released)
 20           amd64 running linux (x86-64/GMP-4.2.2 kernel) 64-bit version
 21          compiled: Nov 30 2007, gcc-3.4.3 20041212 (Red Hat 3.4.3-9.EL4)
 22                  (readline v4.3 enabled, extended help available)
 23
 24                       Copyright (C) 2000-2006 The PARI Group
 25
 26  PARI/GP is free software, covered by the GNU General Public License, and
 27  comes WITHOUT ANY WARRANTY WHATSOEVER.
 28
 29  Type ? for help, \q to quit.
 30  Type ?12 for how to get moral (and possibly technical) support.
 31
 32
 33  -----------------------------------------------------
 34  10 digits
 35  -----------------------------------------------------
 36
 37  ? PintzRuzsa_psiapprox(4/5+10^(-20),13,10)
 38  The expected number of correct decimal digits is = 10
 39  Approximation for the minimal lambda is = 10^(-10)
 40  Needed matrix exponent for this precision is = 2^39
 41  Number of iterations in the dyadic procedure = 162
 42  The approximated final values are:
 43  max-upper-matrix = 0.9123781030527322323638
```

```
44 │ min-upper-matrix = 0.9123781030521349899098
45 │ max-lower-matrix = 0.9123781030527020894117
46 │ min-lower-matrix = 0.9123781030521048469576
47 │ The approximated values for the moments are:
48 │ moment-max-upper-matrix = 0.6181690752736390706245
49 │ moment-min-upper-matrix = 0.6181690752728714301565
50 │ moment-max-lower-matrix = 0.6181690752736003276488
51 │ moment-min-lower-matrix = 0.6181690752728326871808
52 │ The minimal lambda is in [1.285307939545900000000,1.285307939546100000000]
53 │ Final result (in the centre of the interval): d = 0.9123781030527322323638
54 │ time = 3,459 ms.
55 │
56 │
57 │ ------------------------------------------------------
58 │ 20 digits
59 │ ------------------------------------------------------
60 │
61 │ ? PintzRuzsa_psiapprox(4/5+10^(-20),20,20)
62 │ The expected number of correct decimal digits is = 20
63 │ Approximation for the minimal lambda is = 10^(-20)
64 │ Needed matrix exponent for this precision is = 2^72
65 │ Number of iterations in the dyadic procedure = 286
66 │ The approximated final values are:
67 │ max-upper-matrix = 0.9123781030527583497213405612(1053
68 │ min-upper-matrix = 0.9123781030527583497212710330(3804
69 │ max-lower-matrix = 0.9123781030527583497213382830(0078
70 │ min-lower-matrix = 0.9123781030527583497212687548(2828
71 │ The approximated values for the moments are:
72 │ moment-max-upper-matrix = 0.6181690752661886272476912721(0477
73 │ moment-min-upper-matrix = 0.6181690752661886272476019069(9264
74 │ moment-max-lower-matrix = 0.6181690752661886272476883439(0369
75 │ moment-min-lower-matrix = 0.6181690752661886272475989787(9155
76 │ The minimal lambda is in [1.285307939537797246997990000000,1.285307939537797246998010000000]
77 │ Final result (in the centre of the interval): d = 0.9123781030527583497213405612(1053
78 │ time = 40,779 ms.
79 │
80 │ ------------------------------------------------------
81 │ 30 digits
82 │ ------------------------------------------------------
83 │
84 │ ? PintzRuzsa_psiapprox(4/5+10^(-20),27,30)
85 │ The expected number of correct decimal digits is = 30
86 │ Approximation for the minimal lambda is = 10^(-30)
```

```
 87 │ Needed matrix exponent for this precision is = 2^105
 88 │ Number of iterations in the dyadic procedure = 427
 89 │ The approximated final values are:
 90 │ max-upper-matrix = 0.9123781030527583497213585904599910416545890
 91 │ min-upper-matrix = 0.9123781030527583497213585904599902322401313
 92 │ max-lower-matrix = 0.9123781030527583497213585904599910399454390
 93 │ min-lower-matrix = 0.9123781030527583497213585904599902305309813
 94 │ The approximated values for the moments are:
 95 │ moment-max-upper-matrix = 0.6181690752661886269312993581244294777729334
 96 │ moment-min-upper-matrix = 0.6181690752661886269312993581244190742611045
 97 │ moment-max-lower-matrix = 0.6181690752661886269312993581244294557761493
 98 │ moment-min-lower-matrix = 0.6181690752661886269312993581244190522932204
 99 │ The minimal lambda is in [1.285307939537797246651197412280059000000000,1.285307939537797246651197412280061100000000]
100 │ Final result (in the centre of the interval): d = 0.9123781030527583497213585904599910416545890
101 │ time = 3mn, 48,573 ms.
102 │
103 │ -------------------------------------------------------
104 │ 50 digits
105 │ -------------------------------------------------------
106 │
107 │ ? PintzRuzsa_psiapprox(4/5+10^(-20),39,50)
108 │ The expected number of correct decimal digits is = 50
109 │ Approximation for the minimal lambda is = 10^(-50)
110 │ Needed matrix exponent for this precision is = 2^172
111 │ Number of iterations in the dyadic procedure = 642
112 │ The approximated final values are:
113 │ max-upper-matrix = 0.9123781030527583497213585904599092941408561524467593011308271
114 │ min-upper-matrix = 0.9123781030527583497213585904599092941408561524467592482650256 1
115 │ max-lower-matrix = 0.9123781030527583497213585904599092941408561524467593008170857
116 │ min-lower-matrix = 0.9123781030527583497213585904599092941408561524467592482336514 8
117 │ The approximated values for the moments are:
118 │ moment-max-upper-matrix = 0.6181690752661886269312993580825389293962056240740524616375639 4
119 │ moment-min-upper-matrix = 0.6181690752661886269312993580825389293962056240740523911409206 9
120 │ moment-max-lower-matrix = 0.6181690752661886269312993580825389293962056240740524615972385 2
121 │ moment-min-lower-matrix = 0.6181690752661886269312993580825389293962056240740523911005952 7
122 │ The minimal lambda is in [1.28530793953779724665119741223414799755620117815144599000000000,1.28530793953779724665119741223414799755 6
123 │ Final result (in the centre of the interval): d = 0.9123781030527583497213585904599092941408561524467593011308271
124 │ ? ##
125 │   ***   last result computed in 29mn, 1,394 ms.
126 │ -------------------------------------------------------
127 │
128 │ -------------------------------------------------------
129 │ 2/3 on a quad core pc
```

```
130  -------------------------------------------------------
131  [languasc@labsrv0 ~]$ nice /usr/local/Gruppi/PariGP/bin/gp2c-run -pmy_ -g -W PRmethod-KB.gp
132  PRmethod-KB.gp.c: In function `my_PintzRuzsa':
133  PRmethod-KB.gp.c:107: warning: unused variable `my_j'
134  PRmethod-KB.gp.c:107: warning: unused variable `my_l'
135  PRmethod-KB.gp.c:107: warning: unused variable `my_i'
136  PRmethod-KB.gp.c:107: warning: unused variable `my_i1'
137  PRmethod-KB.gp.c:107: warning: unused variable `my_i2'
138                   GP/PARI CALCULATOR Version 2.3.2 (released)
139          amd64 running linux (x86-64/GMP-4.2.2 kernel) 64-bit version
140        compiled: Nov 30 2007, gcc-3.4.3 20041212 (Red Hat 3.4.3-9.EL4)
141                 (readline v4.3 enabled, extended help available)
142
143                     Copyright (C) 2000-2006 The PARI Group
144
145  PARI/GP is free software, covered by the GNU General Public License, and
146  comes WITHOUT ANY WARRANTY WHATSOEVER.
147
148  Type ? for help, \q to quit.
149  Type ?12 for how to get moral (and possibly technical) support.
150
151  -------------------------------------------------------
152  10 digits
153  -------------------------------------------------------
154
155  ? PintzRuzsa_psiapprox(2/3+10^(-20),12,10)
156  The expected number of correct decimal digits is = 10
157  Approximation for the minimal lambda is = 10^(-10)
158  Needed matrix exponent for this precision is = 2^39
159  Number of iterations in the dyadic procedure = 160
160  The approximated final values are:
161  max-upper-matrix = 0.8337213168426473838898
162  min-upper-matrix = 0.8337213168420918764748
163  max-lower-matrix = 0.8337213168425383672048
164  min-lower-matrix = 0.8337213168419828597898
165  The approximated values for the moments are:
166  moment-max-upper-matrix = 0.4413015839592436256564
167  moment-min-upper-matrix = 0.4413015839586416916237
168  moment-max-lower-matrix = 0.4413015839591254978852
169  moment-min-lower-matrix = 0.4413015839585235638525
170  The minimal lambda is in [1.083575154049900000000,1.083575154050100000000]
171  Final result (in the centre of the interval): d = 0.8337213168426473838898
172  time = 2,774 ms.
```

```
173
174  -------------------------------------------------------
175  20 digits
176  -------------------------------------------------------
177
178  ? PintzRuzsa_psiapprox(2/3+10^(-20),19,20)
179  The expected number of correct decimal digits is = 20
180  Approximation for the minimal lambda is = 10^(-20)
181  Needed matrix exponent for this precision is = 2^72
182  Number of iterations in the dyadic procedure = 281
183  The approximated final values are:
184  max-upper-matrix = 0.83372131684338485515459227435255
185  min-upper-matrix = 0.83372131684338485515452760477806
186  max-lower-matrix = 0.83372131684338485515458851235650
187  min-lower-matrix = 0.83372131684338485515452384278202
188  The approximated values for the moments are:
189  moment-max-upper-matrix = 0.44130158394251203320300447723651
190  moment-min-upper-matrix = 0.44130158394251203320293440289238
191  moment-max-lower-matrix = 0.44130158394251203320300040083107
192  moment-min-lower-matrix = 0.44130158394251203320293032648693
193  The minimal lambda is in [1.08357515402897295218599000000000,1.08357515402897295218600100000000]
194  Final result (in the centre of the interval): d = 0.83372131684338485515459227435255
195  time = 34,920 ms.
196
197
198  -------------------------------------------------------
199  30 digits
200  -------------------------------------------------------
201
202
203  ? PintzRuzsa_psiapprox(2/3+10^(-20),26,30)
204  The expected number of correct decimal digits is = 30
205  Approximation for the minimal lambda is = 10^(-30)
206  Needed matrix exponent for this precision is = 2^105
207  Number of iterations in the dyadic procedure = 419
208  The approximated final values are:
209  max-upper-matrix = 0.833721316843384855154592152588205415890590
210  min-upper-matrix = 0.833721316843384855154592152588197887361263
211  max-lower-matrix = 0.833721316843384855154592152588205404464508
212  min-lower-matrix = 0.833721316843384855154592152588197875935182
213  The approximated values for the moments are:
214  moment-max-upper-matrix = 0.441301583942512033013203743647633606807313
215  moment-min-upper-matrix = 0.441301583942512033013203743647625449079988
```

```
216 | moment-max-lower-matrix = 0.44130158394251203301320374364763359442629 5
217 | moment-min-lower-matrix = 0.44130158394251203301320374364762543669897 0
218 | The minimal lambda is in [1.08357515402897295195834526955999900000000,1.083575154028972951958345269560001000000000]
219 | Final result (in the centre of the interval): d = 0.83372131684338485515459215258820541589 0590
220 | time = 3mn, 24,120 ms.
221 |
222 | -------------------------------------------------------
223 | 50 digits
224 | -------------------------------------------------------
225 |
226 | ? PintzRuzsa_psiapprox(2/3+10^(-20),37,50)
227 | The expected number of correct decimal digits is = 50
228 | Approximation for the minimal lambda is = 10^(-50)
229 | Needed matrix exponent for this precision is = 2^172
230 | Number of iterations in the dyadic procedure = 632
231 | The approximated final values are:
232 | max-upper-matrix = 0.83372131684338485515459215258820372013640387733149037455160858
233 | min-upper-matrix = 0.83372131684338485515459215258820372013640387733149032353630951
234 | max-lower-matrix = 0.83372131684338485515459215258820372013640387733149037427262730
235 | min-lower-matrix = 0.83372131684338485515459215258820372013640387733149032325732823
236 | The approximated values for the moments are:
237 | moment-max-upper-matrix = 0.4413015839425120330132037436202609596731541683252580175366240 5
238 | moment-min-upper-matrix = 0.4413015839425120330132037436202609596731541683252579622577135 0
239 | moment-max-lower-matrix = 0.4413015839425120330132037436202609596731541683252580172343268 7
240 | moment-min-lower-matrix = 0.4413015839425120330132037436202609596731541683252579619554163 2
241 | The minimal lambda is in [1.08357515402897295195834526952717031326404619895985790000 0000,1.08357515402897295195834526952717031326 4
242 | Final result (in the centre of the interval): d = 0.83372131684338485515459215258820372013640387733149037455160858
243 | time = 24mn, 37,985 ms.
244 |
245 |
246 |
247 | -------------------------------------------------------
248 | -------------------------------------------------------
```