

**Corso di**  
**ELETTRONICA INDUSTRIALE**

***“Analisi del funzionamento  
continuo del convertitore Buck”***

# Argomenti trattati

- Analisi dei circuiti non lineari con interruttori e diodi
- Convertitore abbassatore di tensione (Buck):  
Analisi del funzionamento continuo  
(**Continuous Conduction Mode, CCM**)

# Argomenti trattati

- **Analisi dei circuiti non lineari con interruttori e diodi**
- **Convertitore abbassatore di tensione (Buck):  
Analisi del funzionamento continuo  
(Continuous Conduction Mode, CCM)**
  - **Fase di on (interruttore chiuso)**
  - **Fase di off (interruttore aperto)**
  - **Forme d'onda complessive**
  - **Caratteristica di controllo**
  - **Ondulazione di corrente e di tensione**

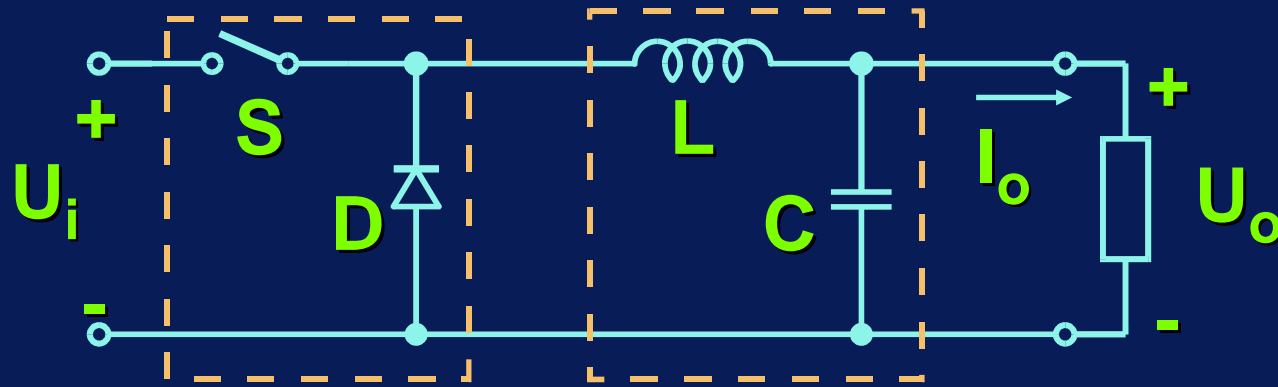
# **Analisi dei circuiti con interruttori**

# **Analisi dei circuiti con interruttori**

## **Approccio lineare a tratti**

- **Si studia separatamente ogni modo di funzionamento (corrispondente ad uno stato di diodi ed interruttori), in cui il circuito é lineare.**
- **Si compongono le sequenze di modi:**
  - **identificando le condizioni di inizio e di fine di ciascun modo**
  - **determinando la successione dei modi**
  - **trasferendo le condizioni finali di un modo come condizioni iniziali del modo seguente**

## Schema del convertitore Buck

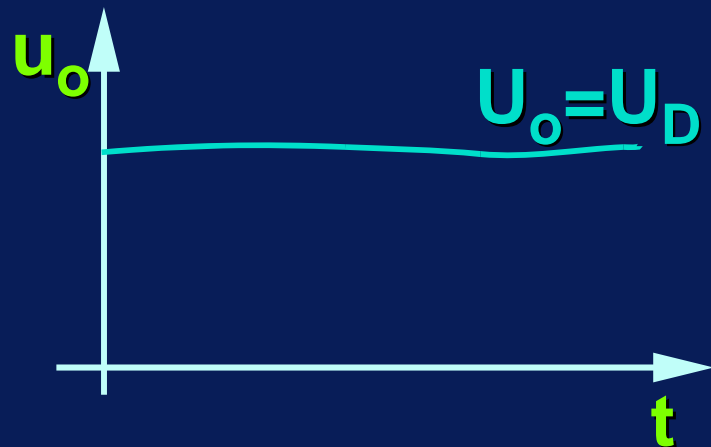


unità switching      filtro

- interruttore ideale ( $u_{S\text{on}}=0, i_{S\text{off}}=0, t_{\text{swon}}=t_{\text{swoff}}=0$ )
- diodo ideale ( $u_{D\text{on}}=0, i_{D\text{off}}=0, t_{\text{swon}}=t_{\text{swoff}}=0$ )
- L,C ideali ( $R_L=0, \text{ESR}=0, \text{ESL}=0$ )
- $u_i = U_i = \text{costante}$
- $u_o = U_o = \text{costante}$       ( $\omega_r \ll 2\pi f_s$ )
- $i_o = I_o = \text{costante}$

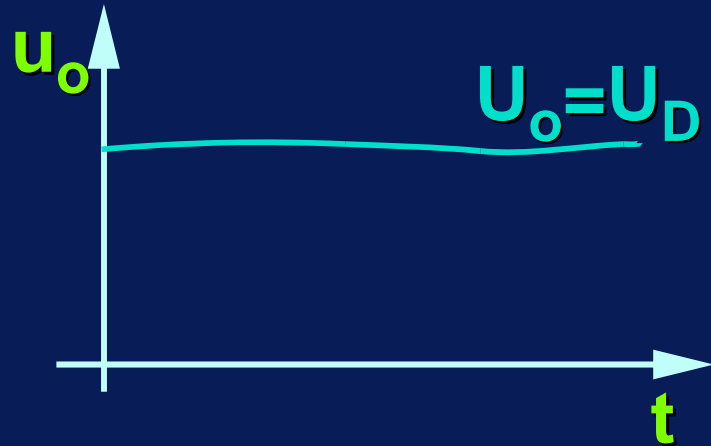
# Forme d'onda tipiche del convertitore

## Forme d'onda tipiche del convertitore



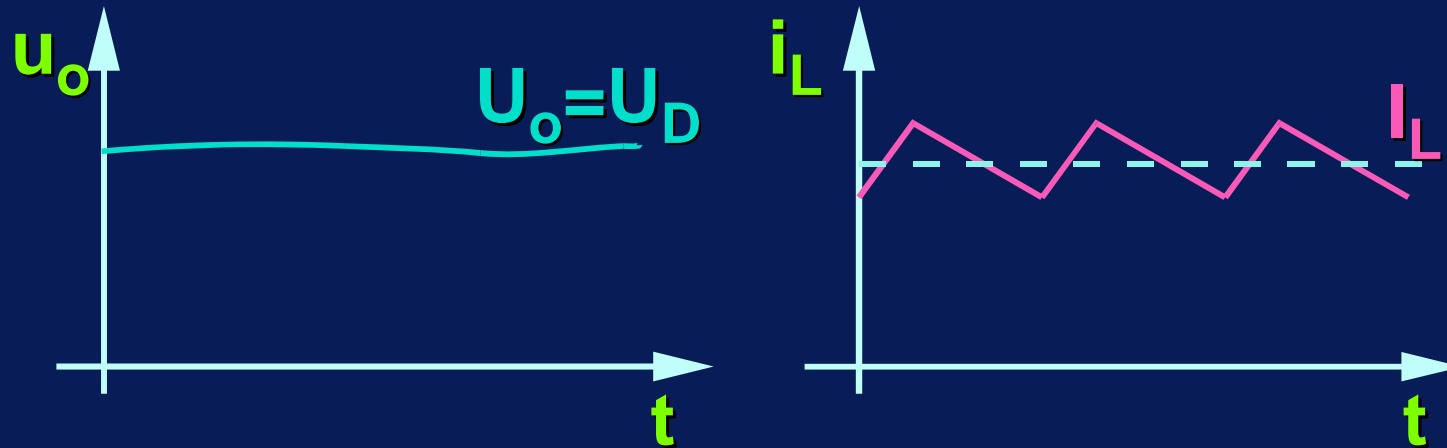


## Forme d'onda tipiche del convertitore



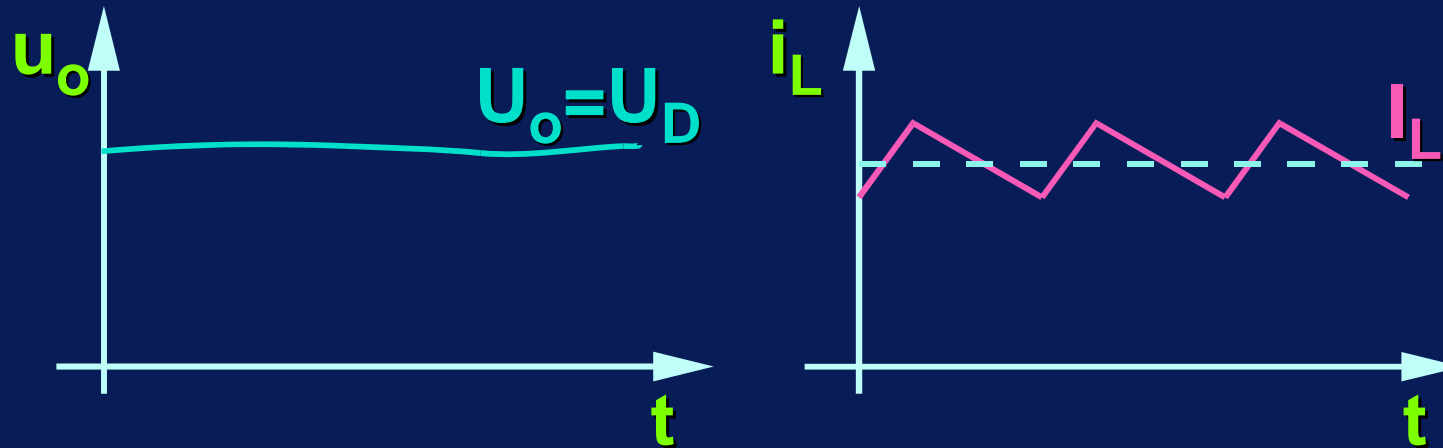
- $u_o$  é effettivamente ben livellata ( $u_o = U_o$ )

## Forme d'onda tipiche del convertitore



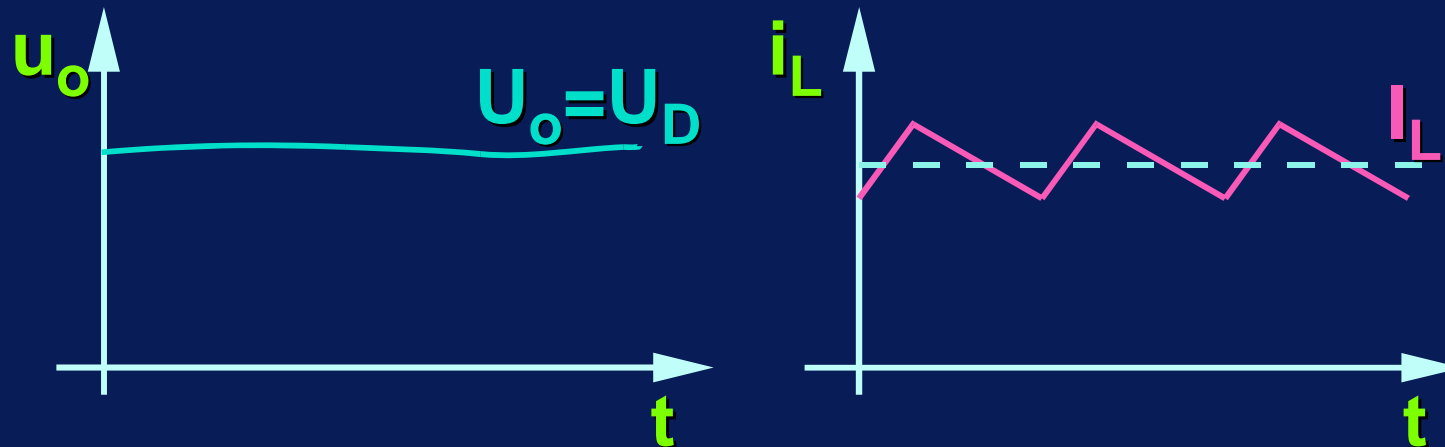
- $u_o$  é effettivamente ben livellata ( $u_o = U_o$ )

## Forme d'onda tipiche del convertitore



- $u_o$  é effettivamente ben livellata ( $u_o = U_o$ )
- $i_L$  ha ondulazione (ripple), ma é sempre  $> 0$

## Forme d'onda tipiche del convertitore



- $u_o$  é effettivamente ben livellata ( $u_o = U_o$ )
- $i_L$  ha ondulazione (ripple), ma é sempre  $> 0$

**Questo modo di funzionamento ( $i_L > 0$ )  
si chiama modo continuo (CCM =  
Continuous Conduction Mode)**

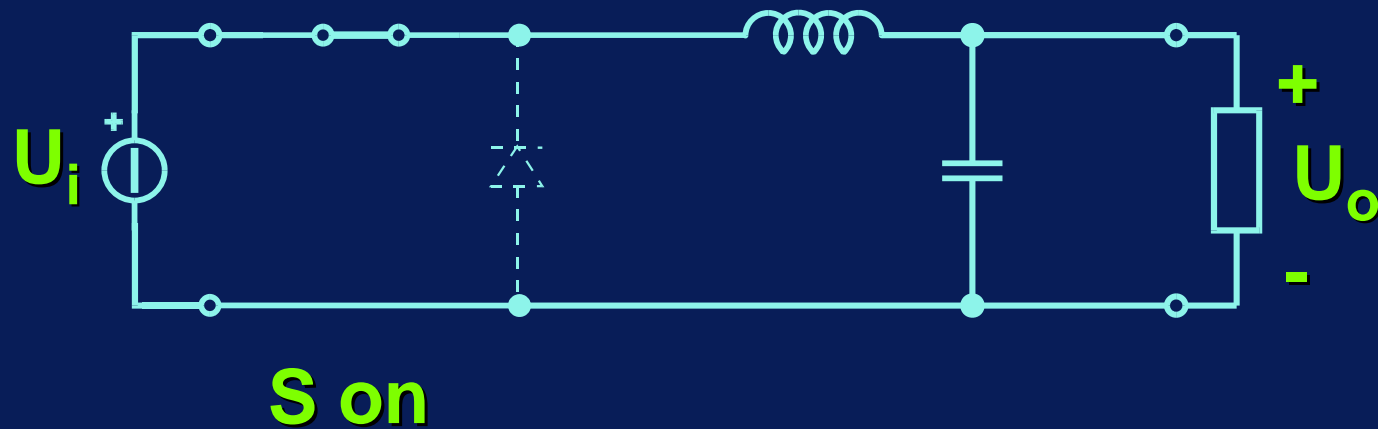
# **Analisi del funzionamento continuo**

# Analisi del funzionamento continuo

Tempo di chiusura di S ( $t_{on}$ )

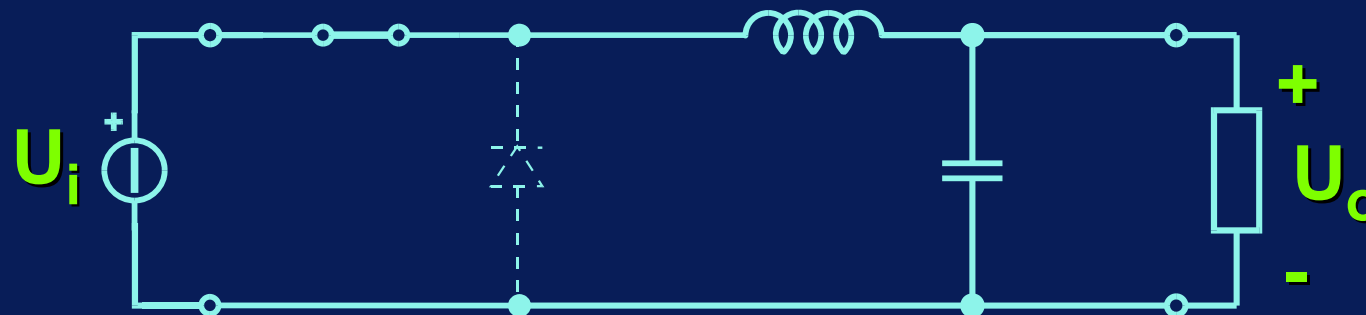
# Analisi del funzionamento continuo

Tempo di chiusura di S ( $t_{on}$ )



# Analisi del funzionamento continuo

Tempo di chiusura di  $S$  ( $t_{on}$ )



**S on D off**

- Il diodo é interdetto.



# Analisi del funzionamento continuo

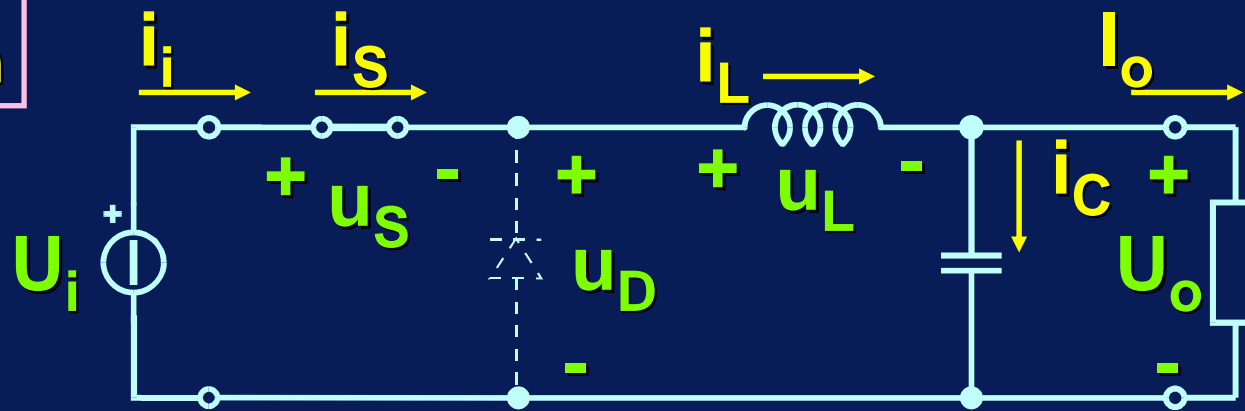
Tempo di chiusura di S ( $t_{on}$ )



**S on D off**

- Il diodo é interdetto.
- Il generatore fornisce energia al filtro e al carico.

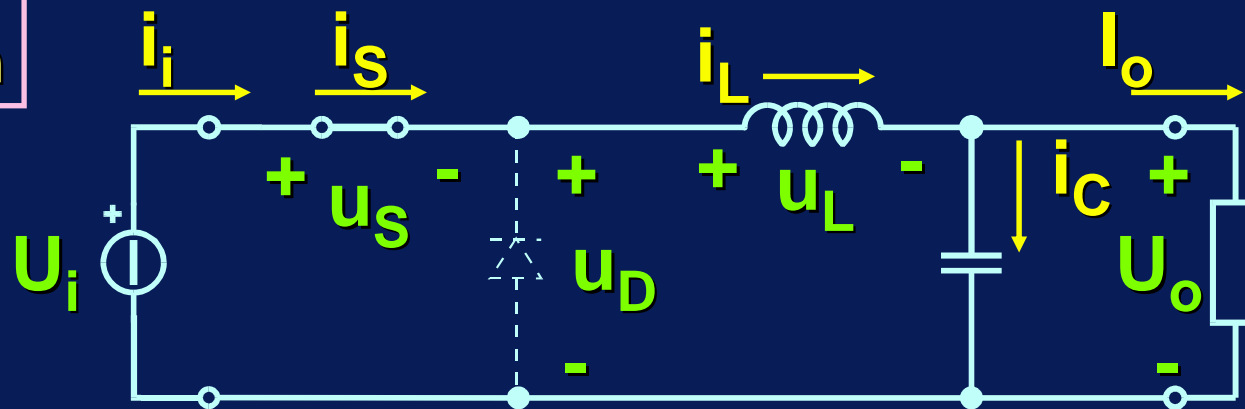
$t_{on}$



S on D off

$$i_i = i_s = i_L = I_o + i_c$$

$t_{on}$

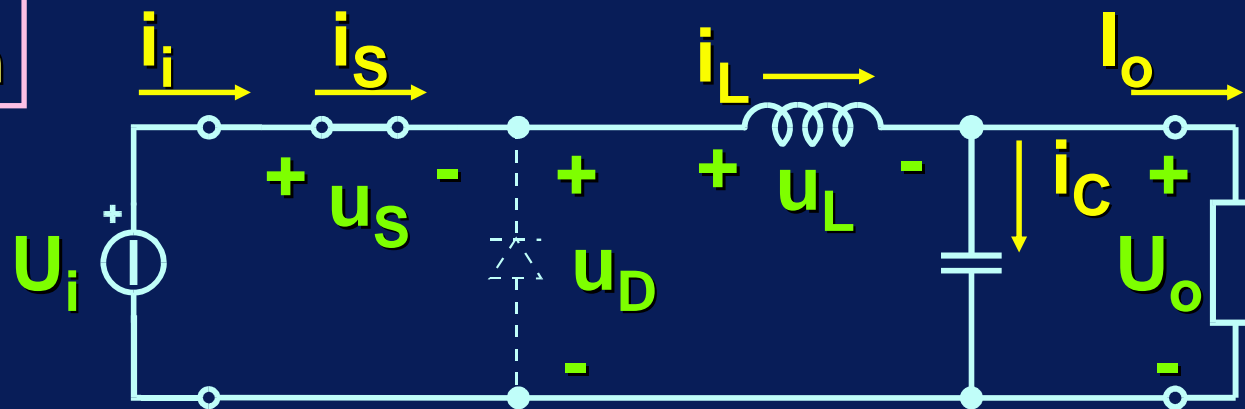


**S on D off**

$$i_i = i_s = i_L = I_o + i_c$$

$$u_D = U_i \quad (\text{diode contropolarizzato})$$

$t_{on}$



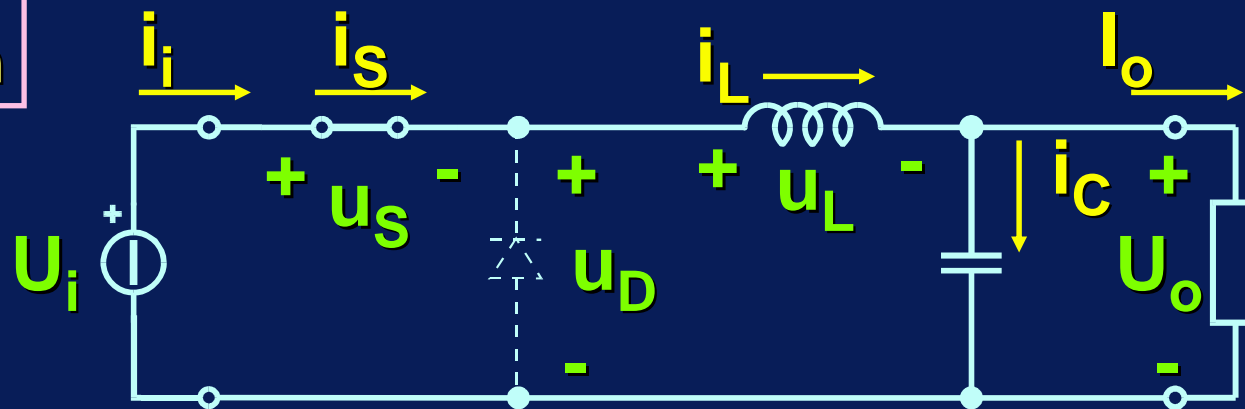
S on D off

$$i_i = i_s = i_L = I_o + i_c$$

$$u_D = U_i \quad (\text{diode contropolarizzato})$$

$$u_L = U_i - U_o$$

$t_{on}$



S on D off

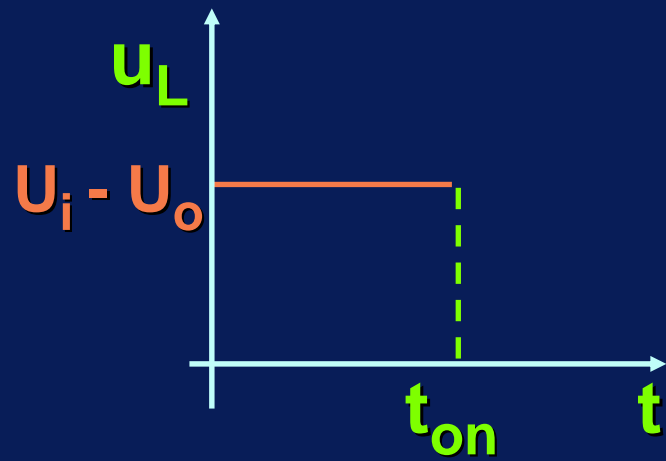
$$i_i = i_s = i_L = I_o + i_c$$

$$u_D = U_i \quad (\text{diode contropolarizzato})$$

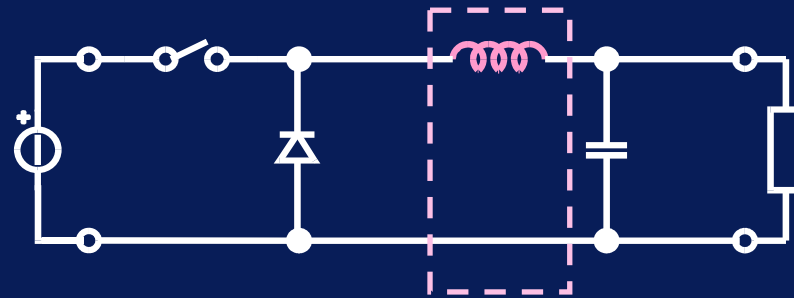
$$u_L = U_i - U_o$$

$$i_L(t) = i_L(0) + \frac{1}{L} \cdot \int_0^t u_L(\tau) d\tau = i_{L \min} + \frac{U_i - U_o}{L} \cdot t$$

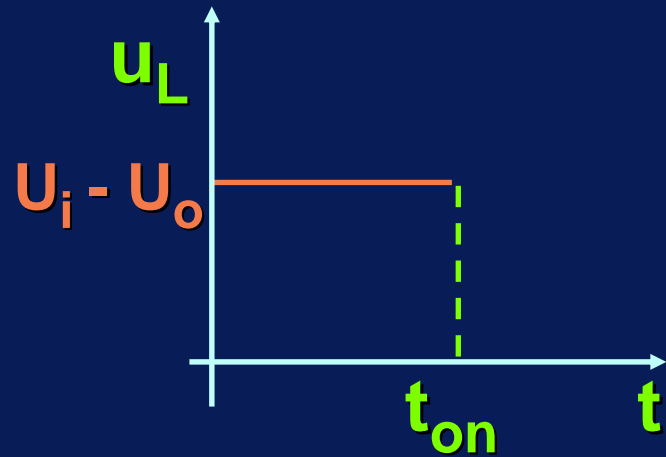
## Tensioni e correnti durante $t_{on}$



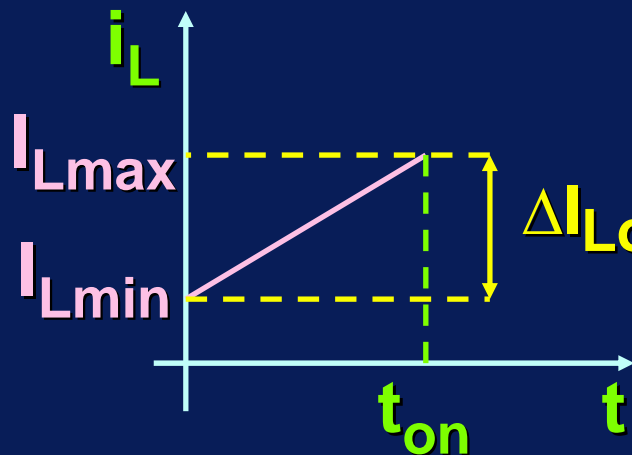
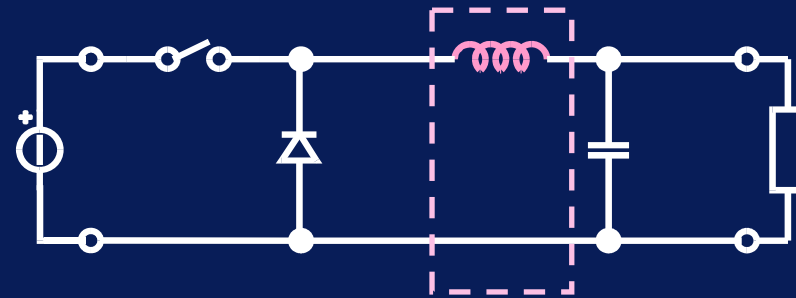
$$u_L = U_i - U_o$$



## Tensioni e correnti durante $t_{on}$

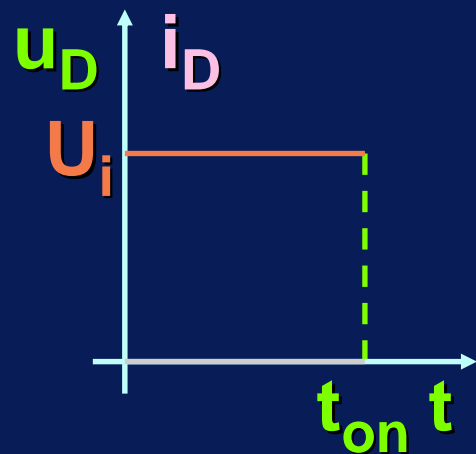


$$u_L = U_i - U_o$$

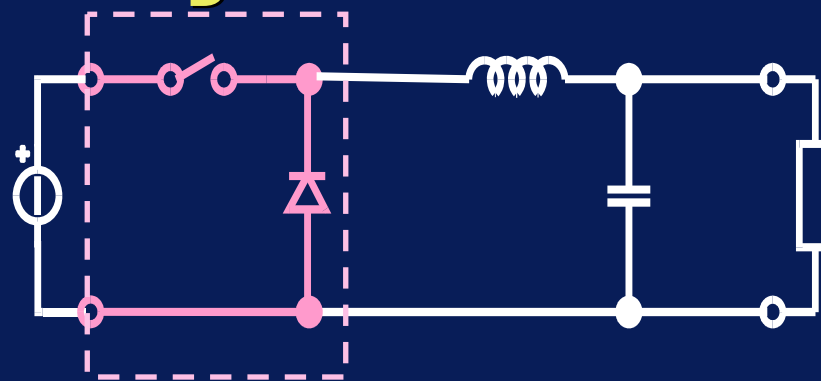


$$\Delta I_{Lon} = \frac{U_i - U_o}{L} \cdot t_{on} = \frac{U_i - U_o}{f_S \cdot L} \cdot \delta$$

## Tensioni e correnti durante $t_{on}$

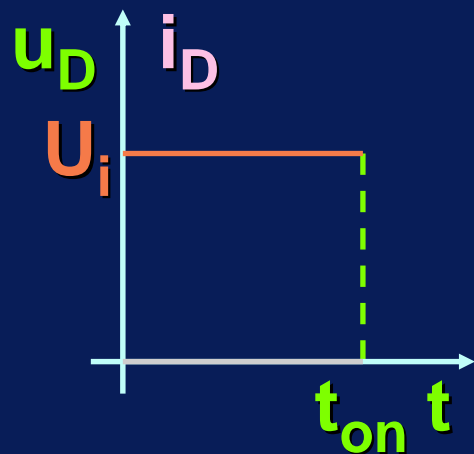


$$u_D = U_i$$
$$i_D = 0$$

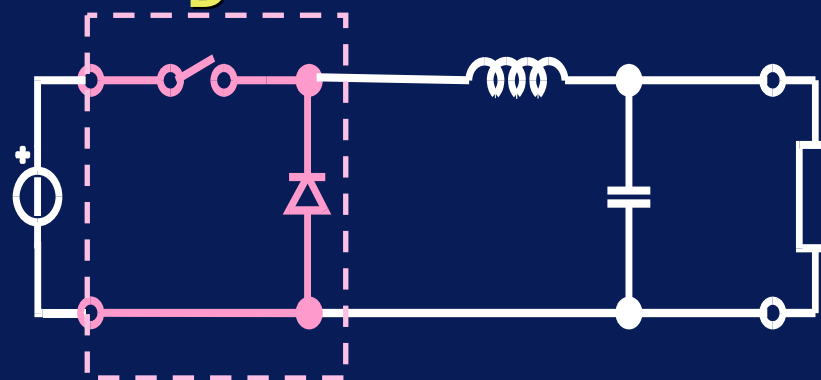




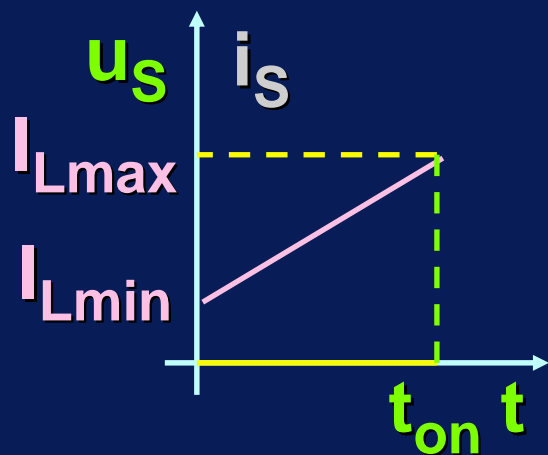
## Tensioni e correnti durante $t_{on}$



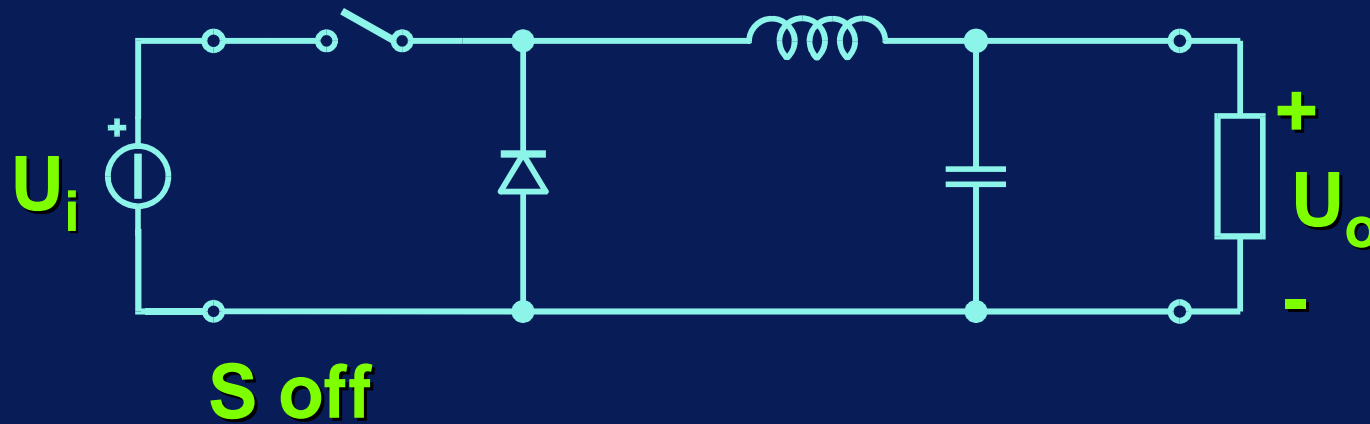
$$u_D = U_i$$
$$i_D = 0$$



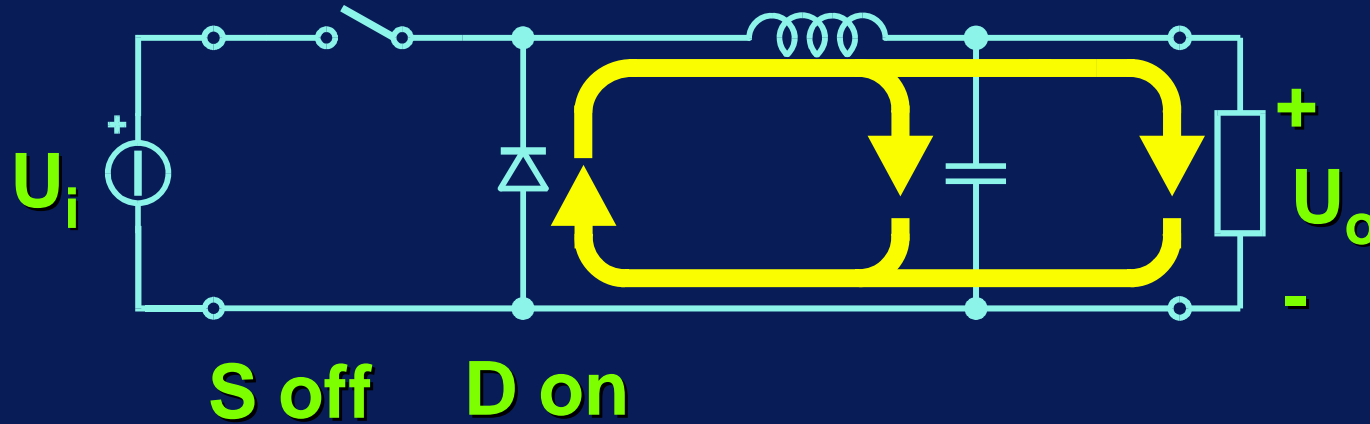
$$u_S = 0$$
$$i_S = i_L$$



## Tempo di apertura di S ( $t_{off}$ )

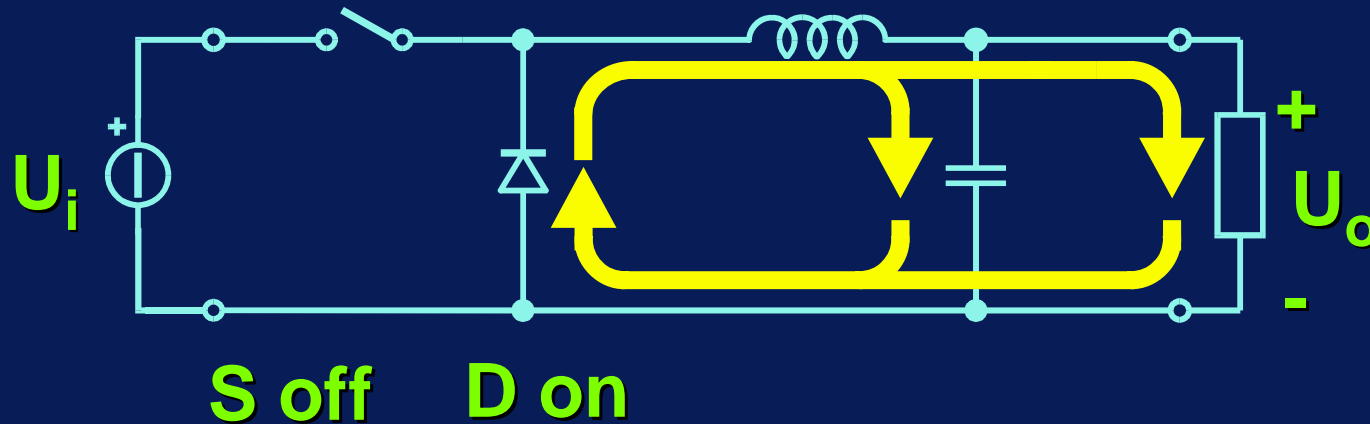


## Tempo di apertura di S ( $t_{off}$ )



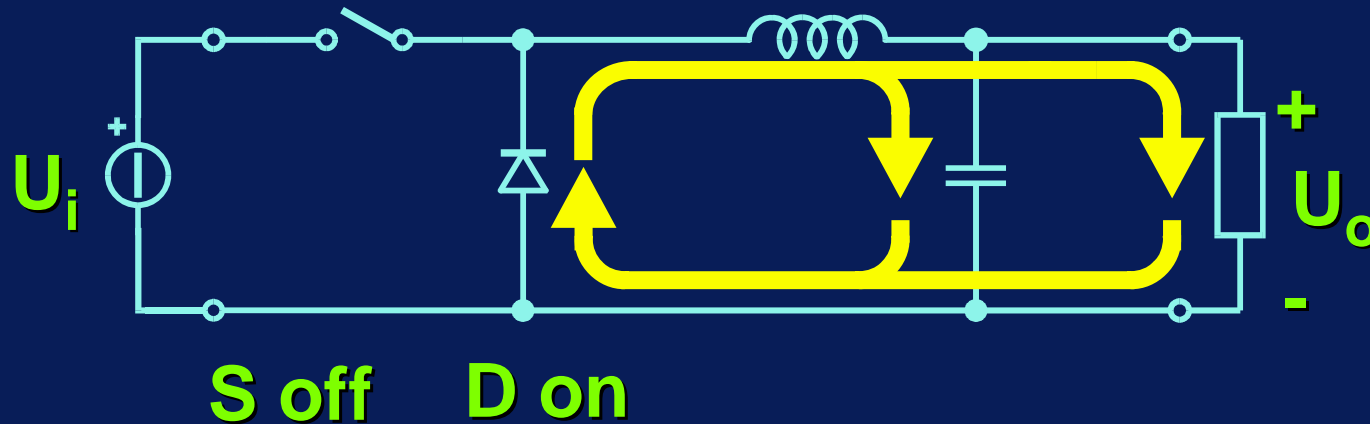
- Il diodo conduce

## Tempo di apertura di S ( $t_{off}$ )



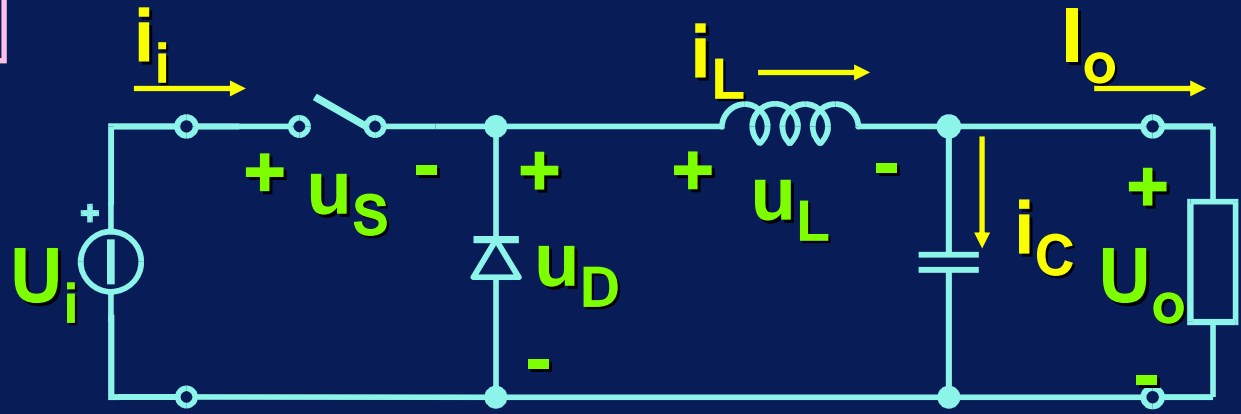
- Il diodo conduce
- L'alimentazione non fornisce energia

## Tempo di apertura di S ( $t_{off}$ )



- Il diodo conduce
- L'alimentazione non fornisce energia
- L'energia del carico viene fornita dal filtro

$t_{\text{off}}$

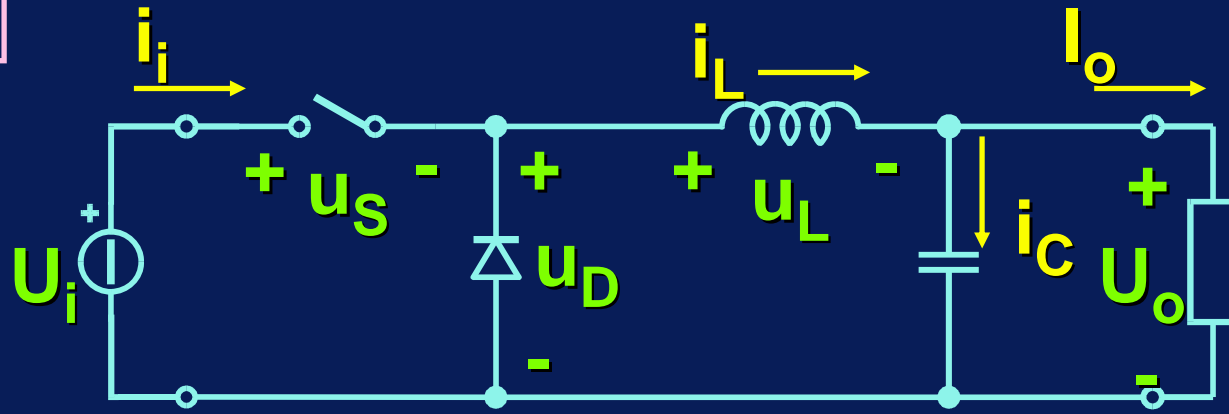


S off D on

$$i_i = 0$$

$$i_L = I_o + i_c$$

$t_{\text{off}}$



S off D on

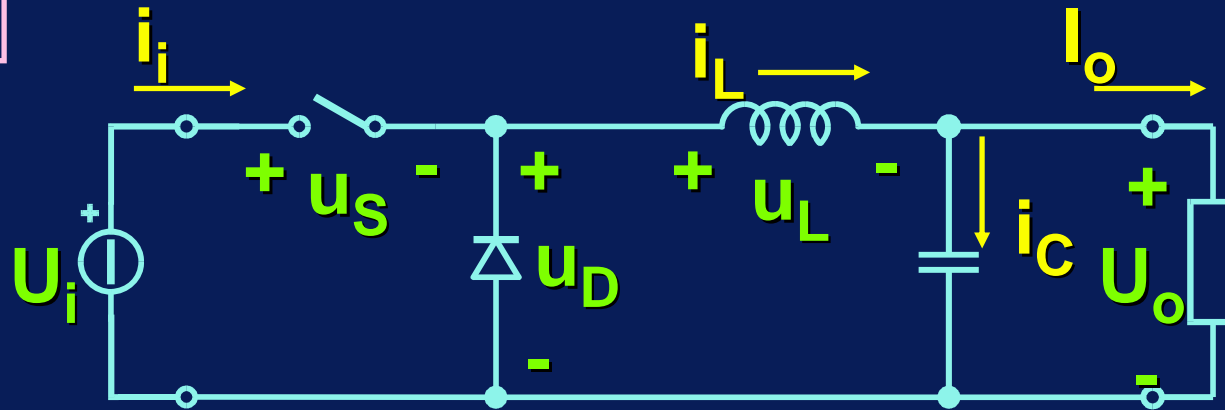
$$i_i = 0$$

$$u_D = 0$$

$$i_L = I_o + i_c$$

$$u_S = U_i$$

$t_{\text{off}}$



**S off D on**

$$i_i = 0$$

$$u_D = 0$$

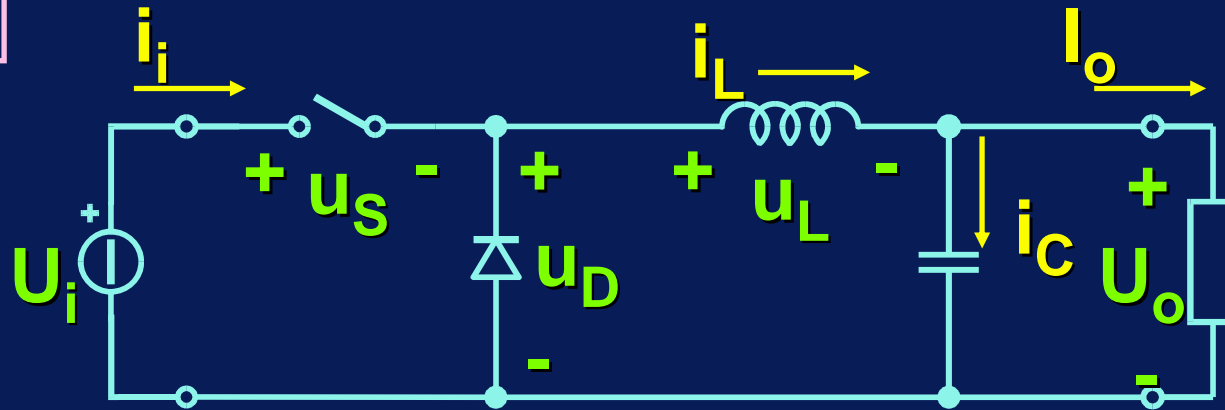
$$u_L = -U_o$$

$$i_L = I_o + i_c$$

$$u_s = U_i$$



$t_{\text{off}}$



**S off    D on**

$$i_i = 0$$

$$i_L = I_o + i_c$$

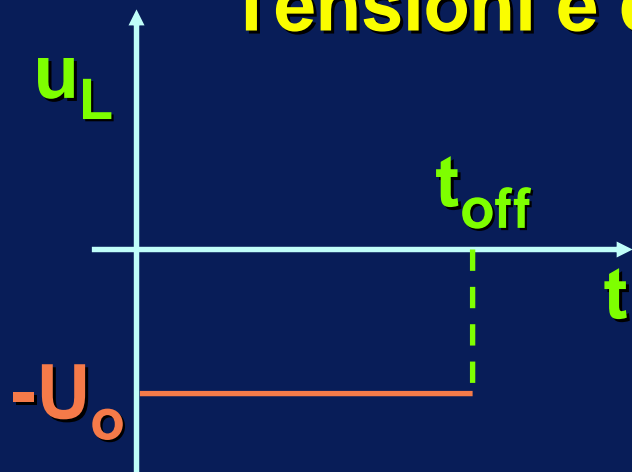
$$u_D = 0$$

$$u_S = U_i$$

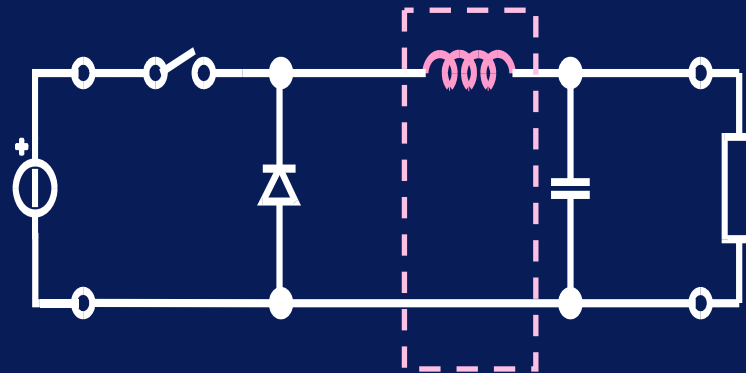
$$u_L = -U_o$$

$$i_L(t) = i_L(0) + \frac{1}{L} \cdot \int_0^t u_L(\tau) d\tau = i_{L \text{ max}} - \frac{U_o}{L} \cdot t$$

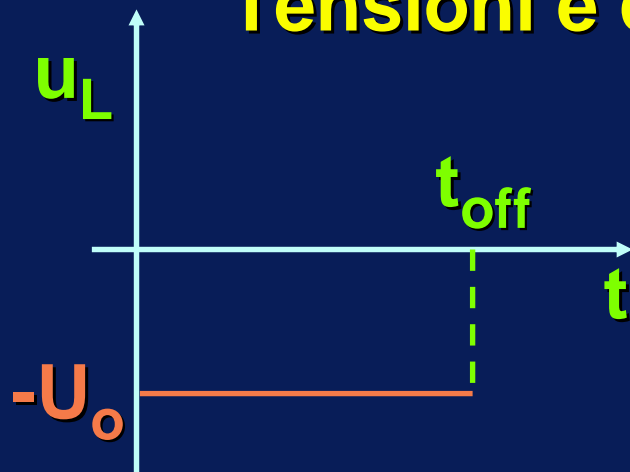
# Tensioni e correnti durante $t_{\text{off}}$



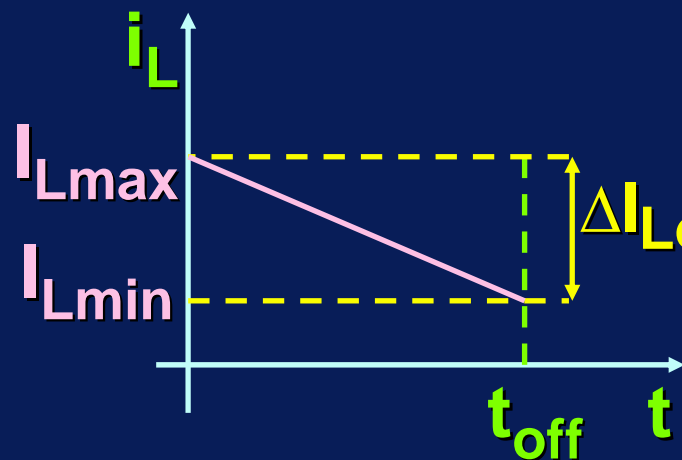
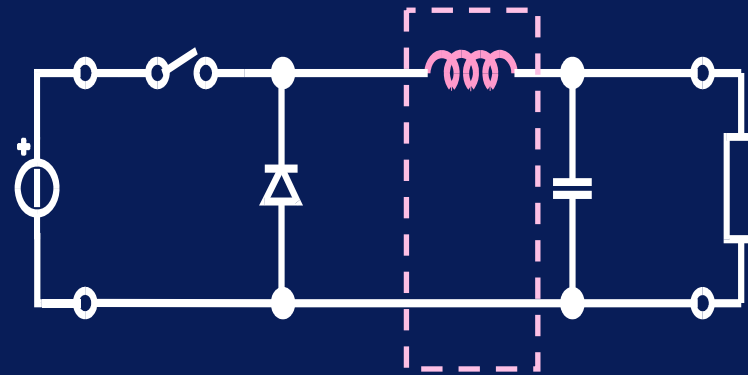
$$u_L = -U_o$$



## Tensioni e correnti durante $t_{\text{off}}$

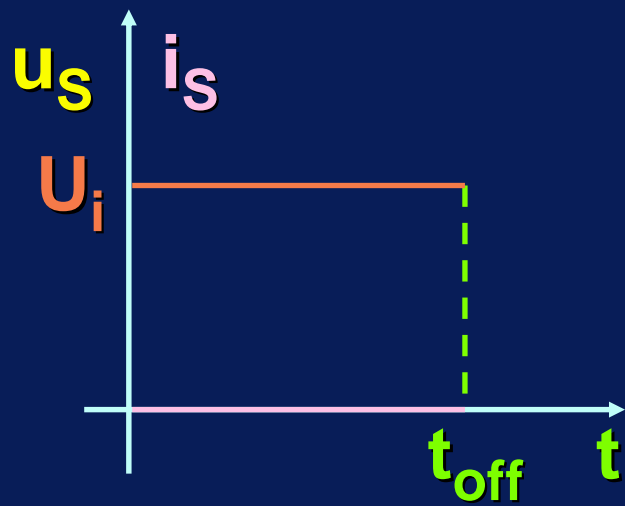


$$u_L = -U_o$$

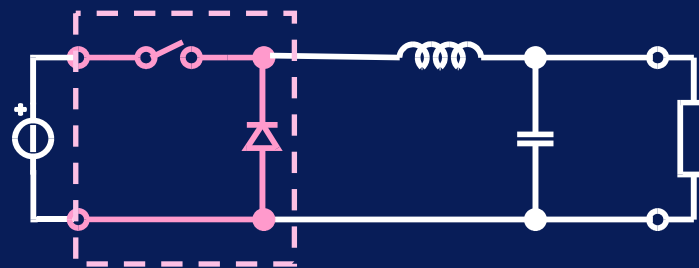


$$\Delta I_{L\text{off}} = \frac{U_o}{L} \cdot t_{\text{off}} = \frac{U_o}{f_S \cdot L} \cdot (1 - \delta)$$

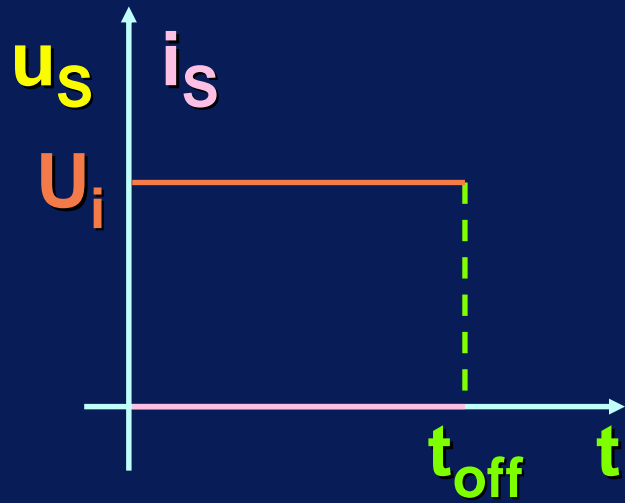
## Tensioni e correnti durante $t_{\text{off}}$



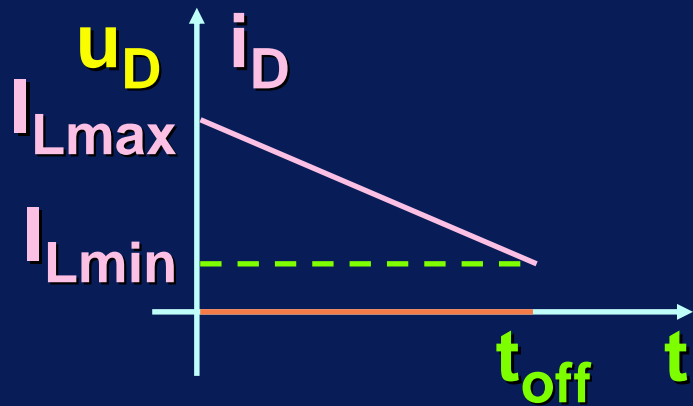
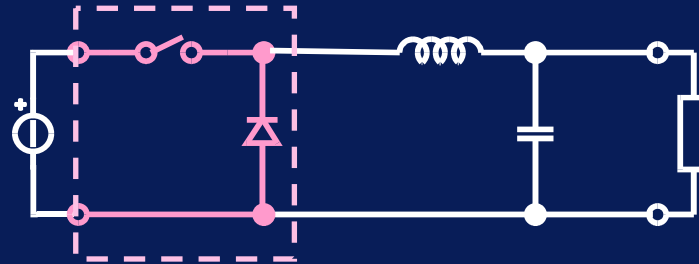
$$u_s = U_i$$
$$i_s = 0$$



## Tensioni e correnti durante $t_{off}$

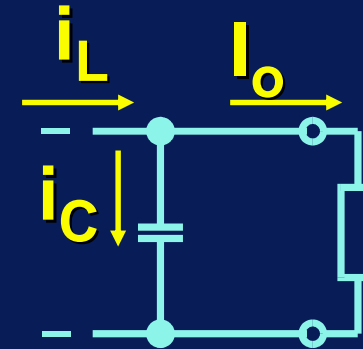
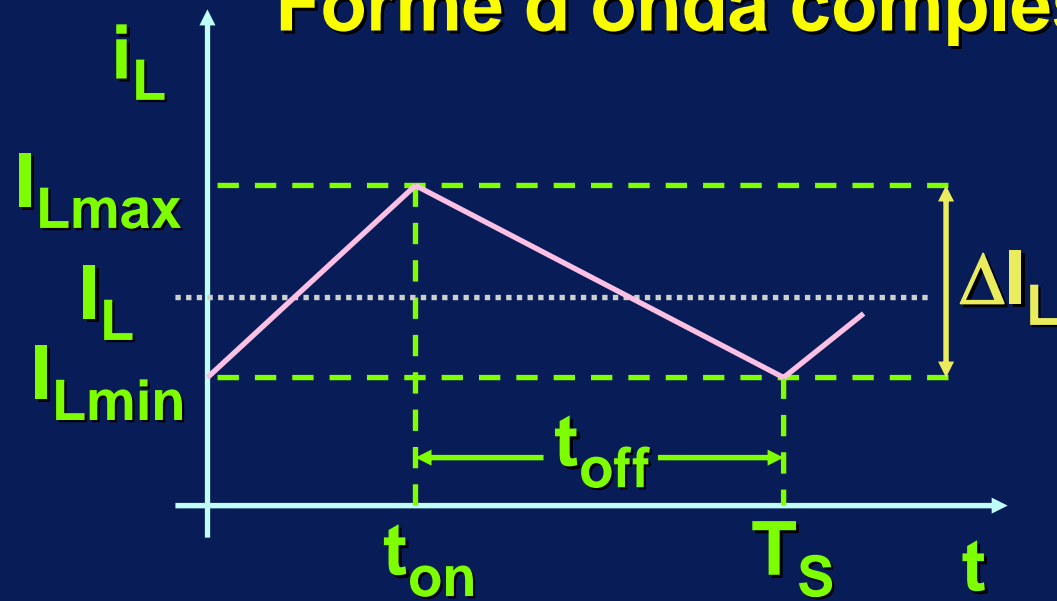


$$u_s = U_i$$
$$i_s = 0$$

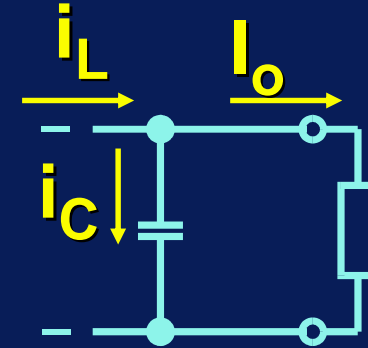
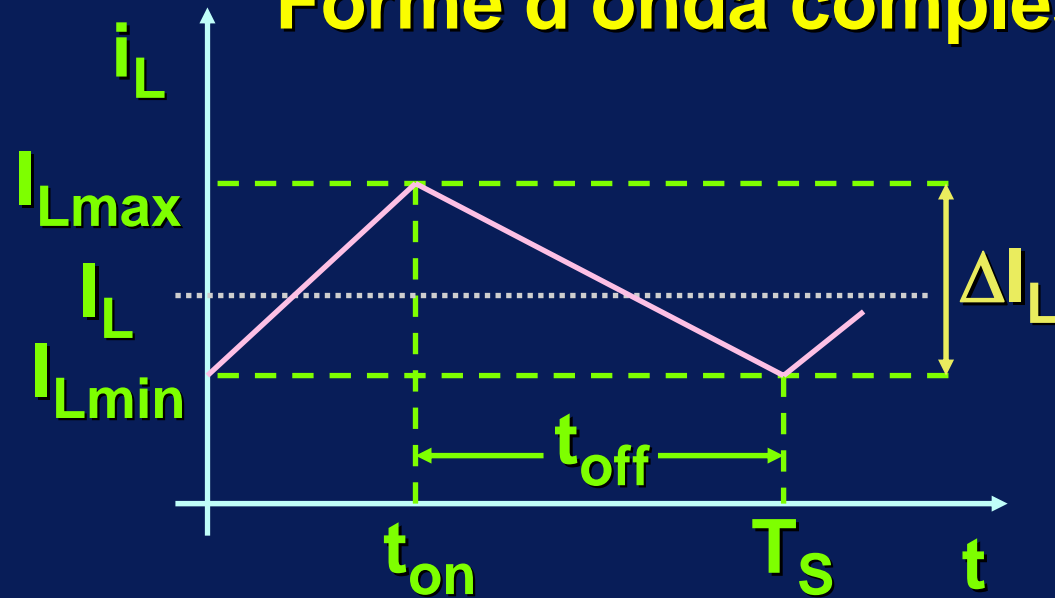


$$u_D = 0$$
$$i_D = i_L$$

## Forme d'onda compressive: $i_L$

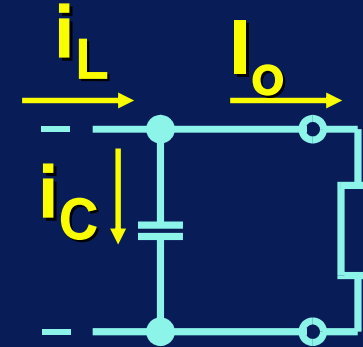
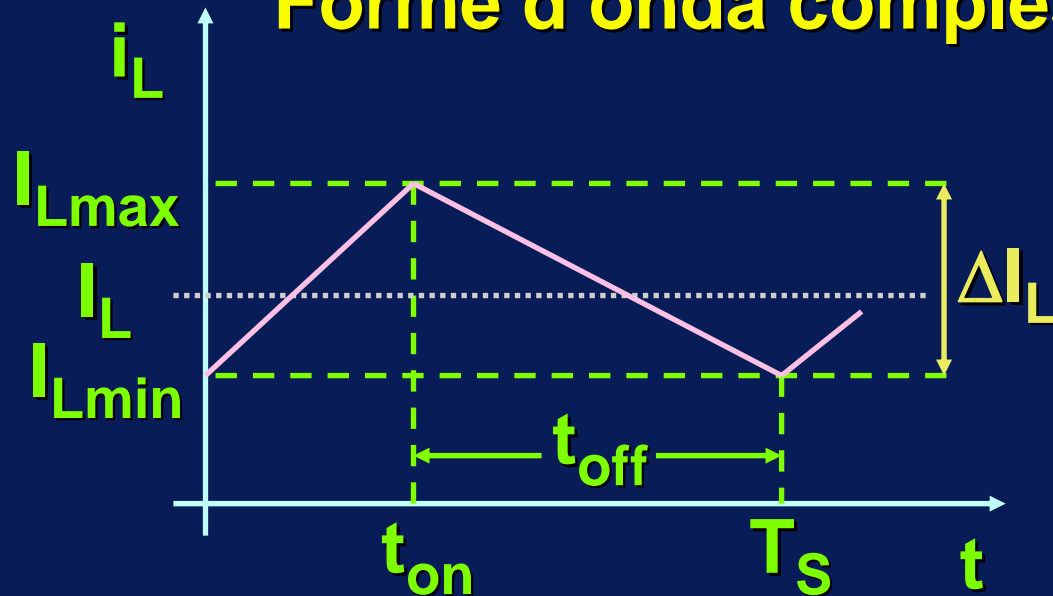


## Forme d'onda compressive: $i_L$



A regime:  $I_C = 0 \Rightarrow I_L = I_o$

## Forme d'onda compressive: $i_L$



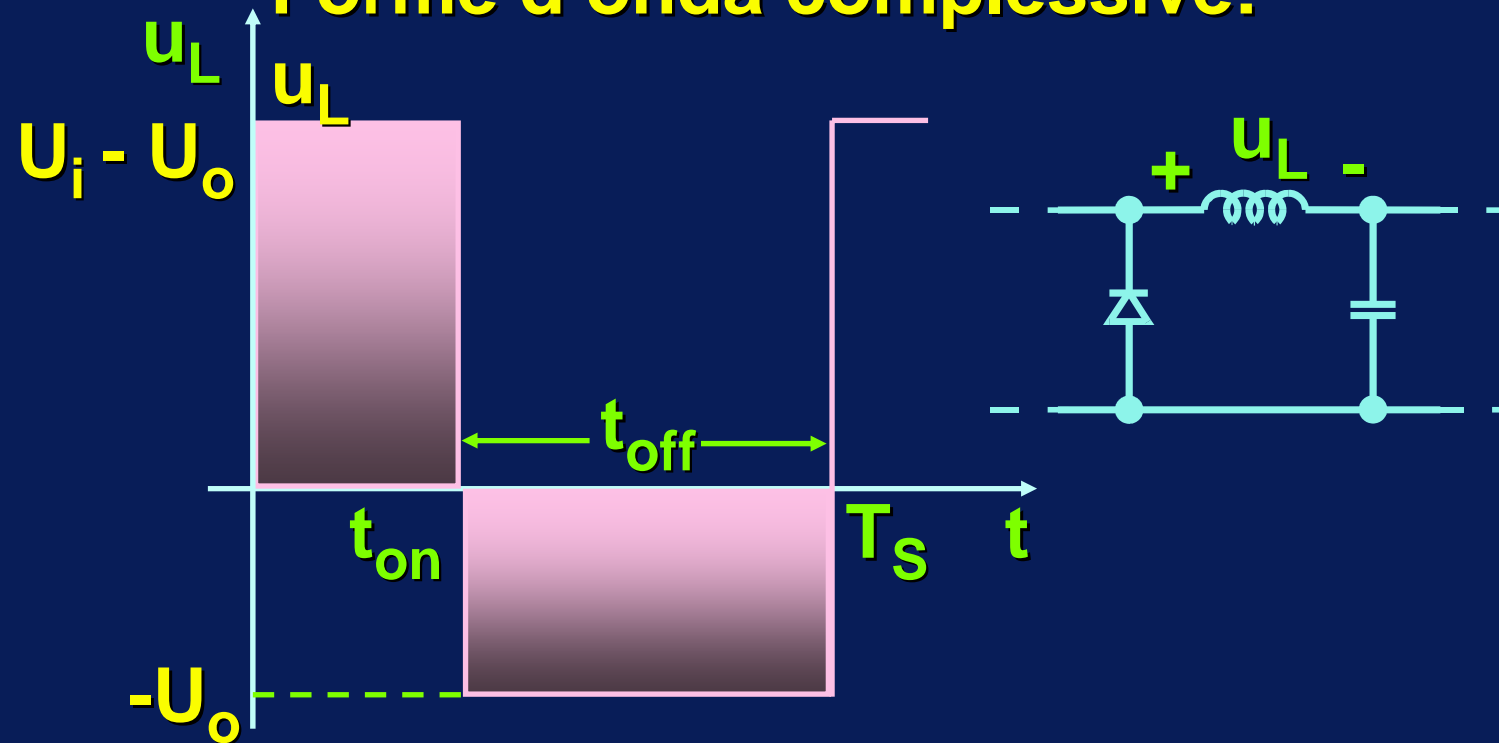
A regime:  $I_C = 0 \Rightarrow I_L = I_o$

Ondulazione (ripple) di corrente:

$$\Delta I_{Lon} = \Delta I_{Loff} = \Delta I_L = \frac{U_i - U_o}{L} \cdot t_{on} = \frac{U_o}{L} \cdot t_{off}$$

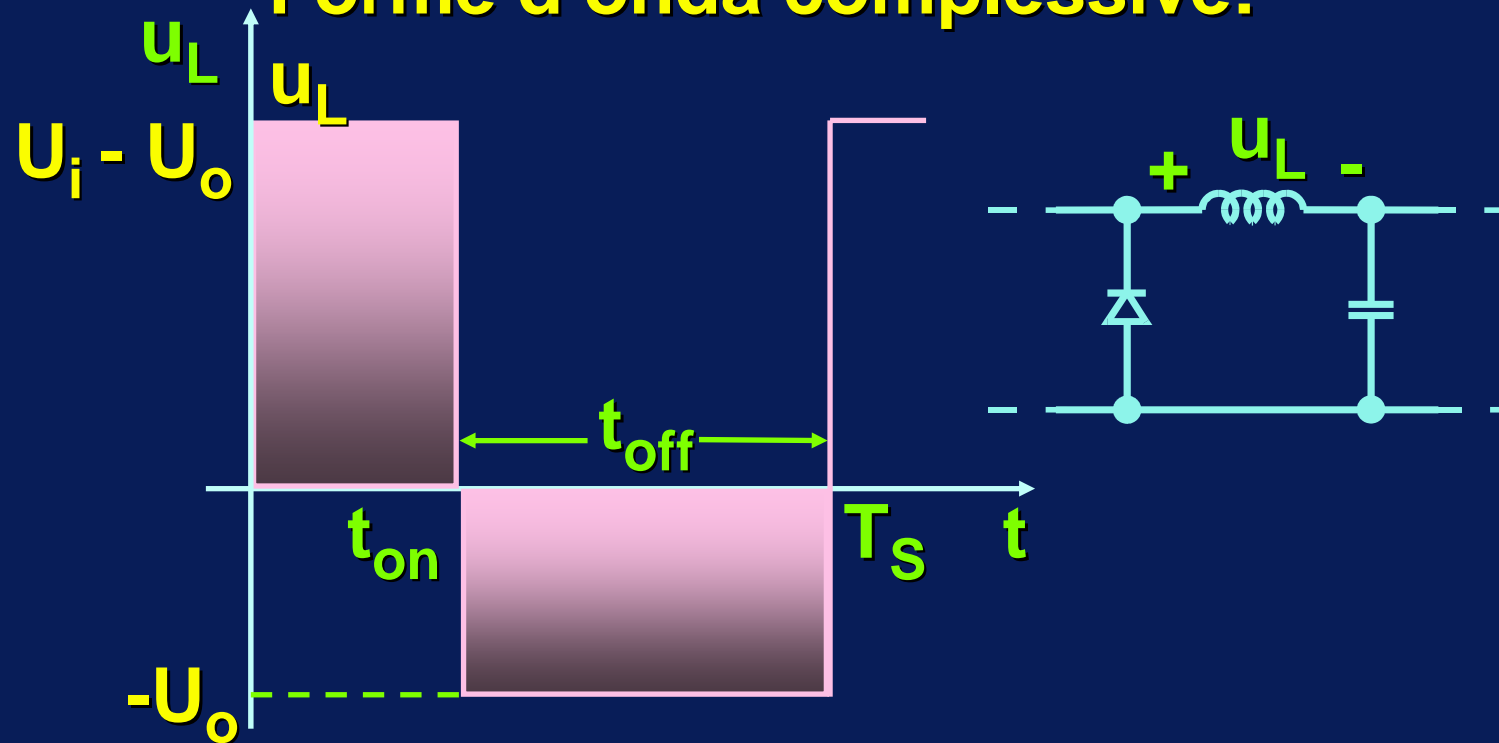


## Forme d'onda compressive:



A regime:  $(U_i - U_o) \cdot t_{on} = U_o \cdot t_{off}$

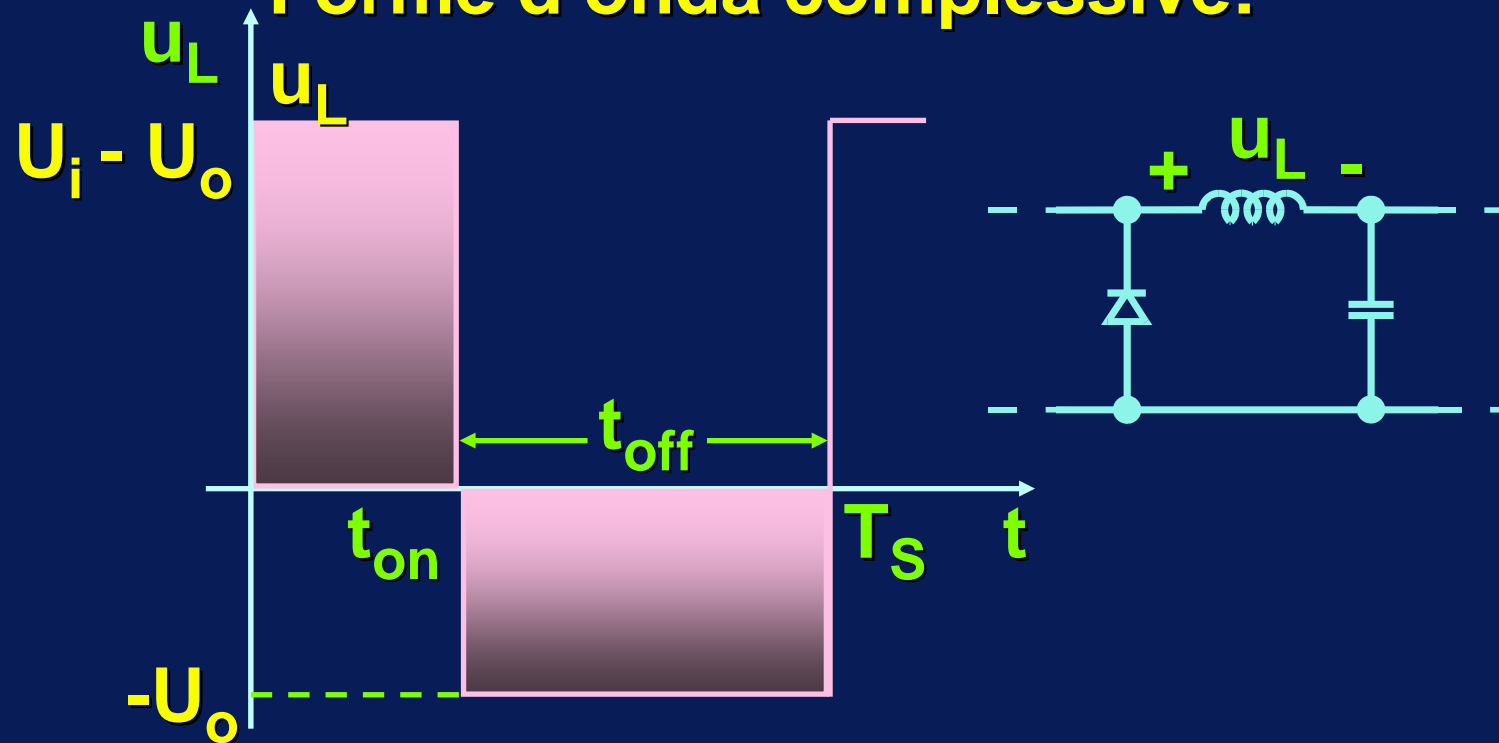
## Forme d'onda compressive:



A regime:  $(U_i - U_o) \cdot t_{on} = U_o \cdot t_{off}$

$$U_i \cdot t_{on} = U_o \cdot (t_{on} + t_{off})$$

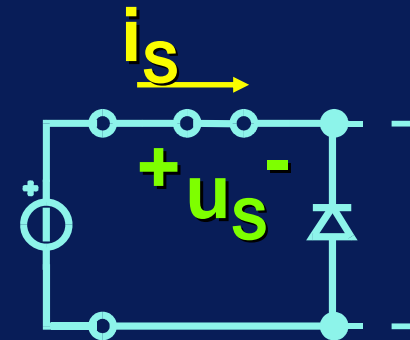
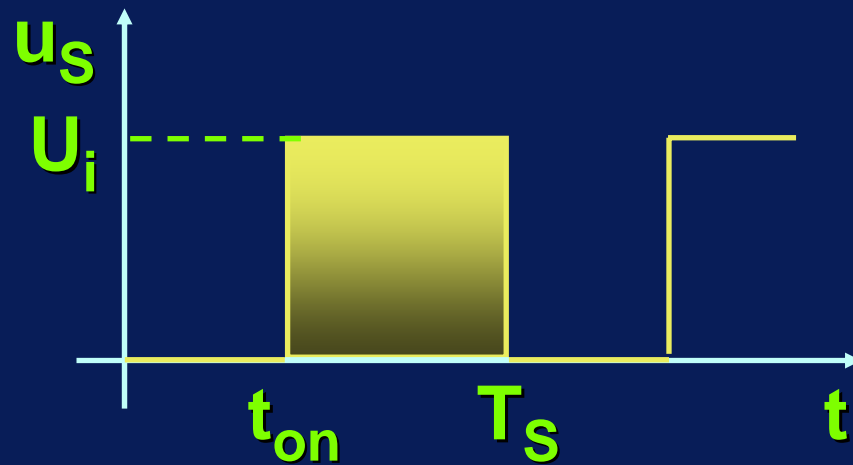
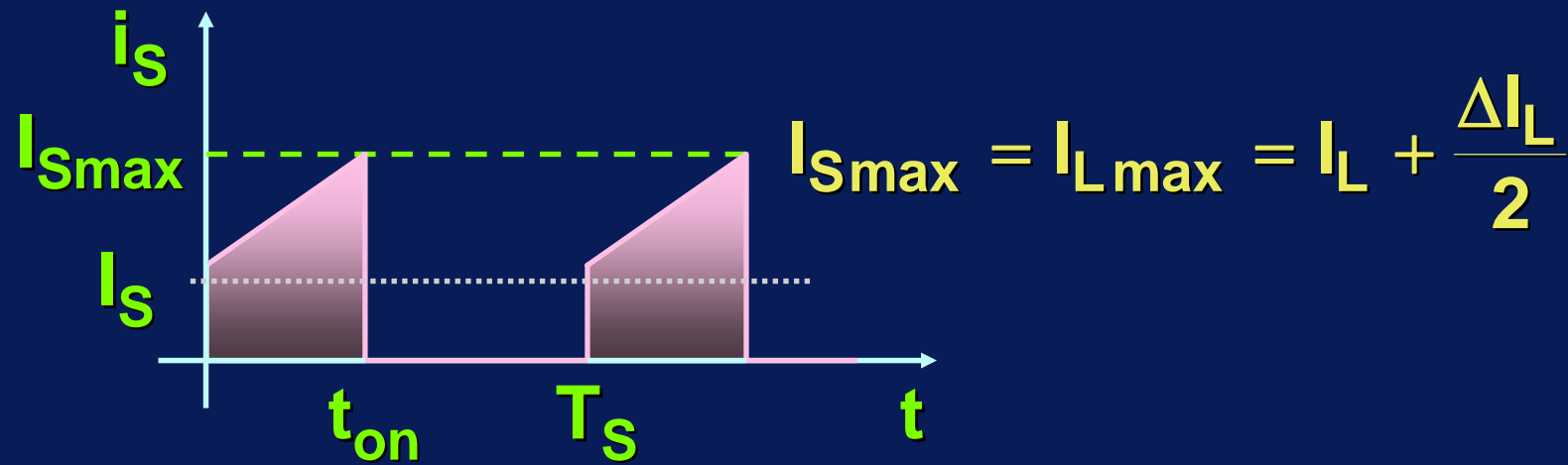
## Forme d'onda compressive:



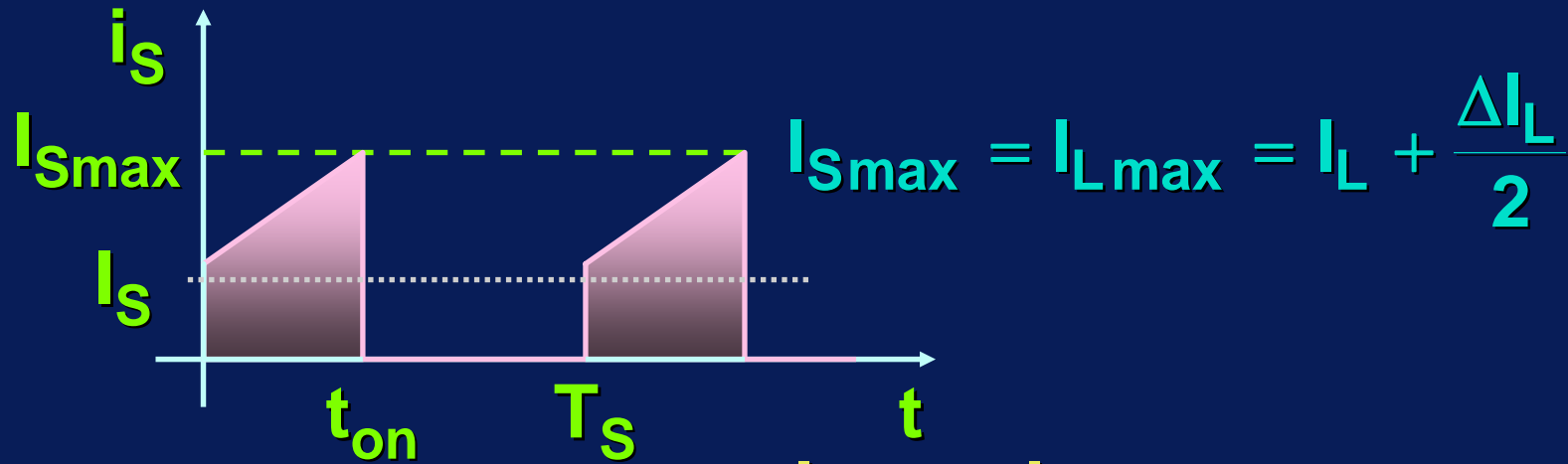
A regime:  $(U_i - U_o) \cdot t_{on} = U_o \cdot t_{off}$

$$U_i \cdot t_{on} = U_o \cdot (t_{on} + t_{off}) \Rightarrow U_o = \frac{t_{on}}{T_s} \cdot U_i = \delta \cdot U_i$$

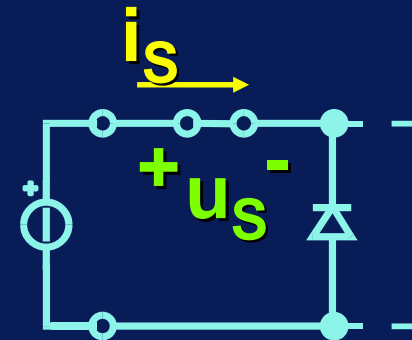
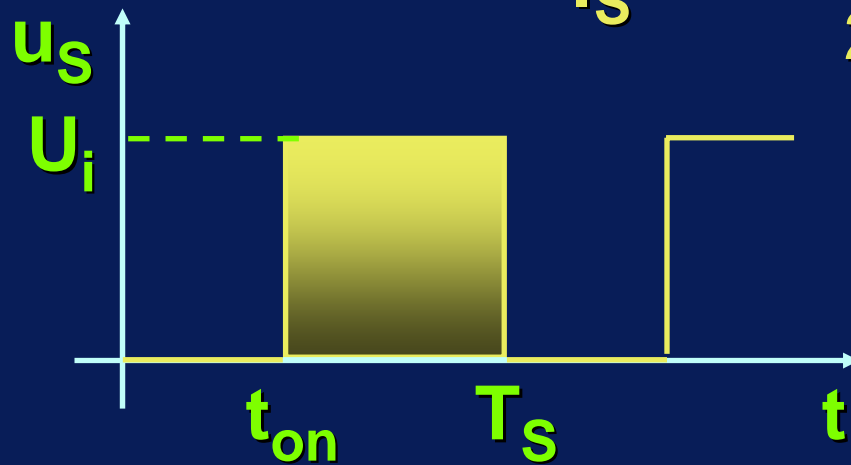
## Forme d'onda compressive: $u_S, i_S$



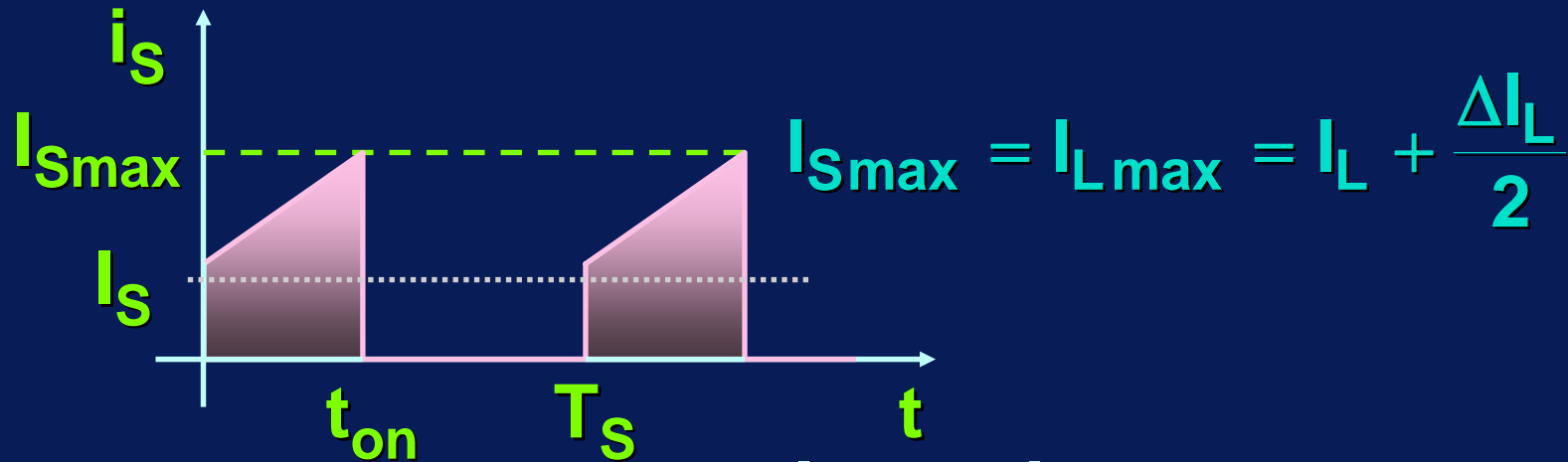
## Forme d'onda compressive: $u_S, i_S$



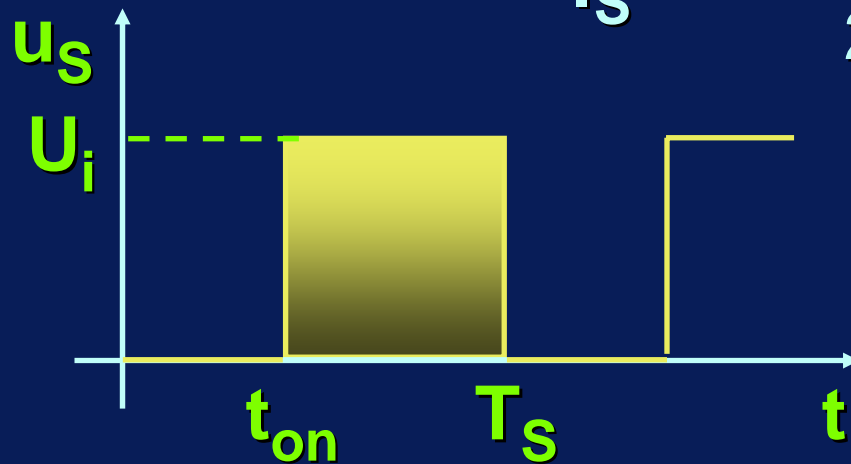
$$I_S = \frac{I_{Lmin} + I_{Lmax}}{2 \cdot T_S} \cdot t_{on} = I_L \cdot \delta$$



## Forme d'onda compressive: $u_s$ , $i_s$



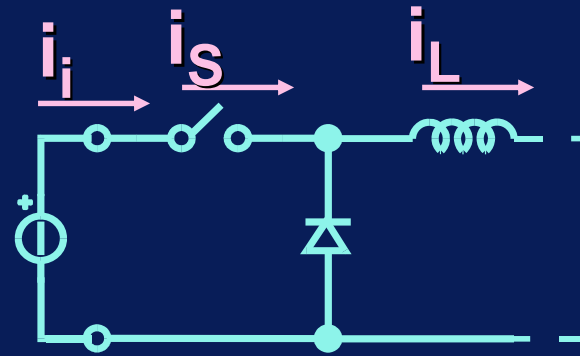
$$I_S = \frac{I_{Lmin} + I_{Lmax}}{2 \cdot T_S} \cdot t_{on} = I_L \cdot \delta$$



$$U_{Smax} = U_i$$

**Nota:**

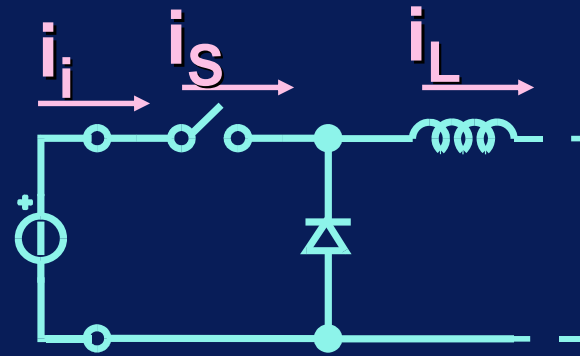
$$I_i = I_s = I_L \cdot \delta = I_o \cdot \delta$$



**Nota:**

$$I_i = I_s = I_L \cdot \delta = I_o \cdot \delta$$

$$P_i = U_i \cdot I_i = U_i \cdot I_o \cdot \delta$$

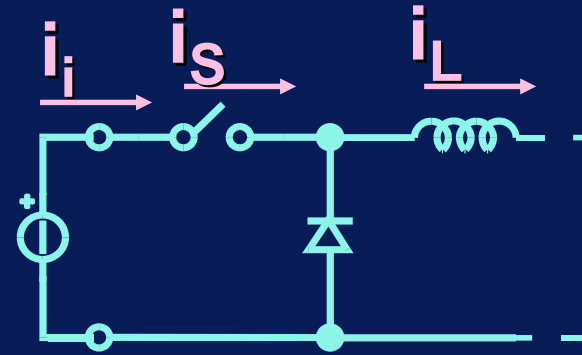




**Nota:**

$$I_i = I_s = I_L \cdot \delta = I_o \cdot \delta$$

$$P_i = U_i \cdot I_i = U_i \cdot I_o \cdot \delta$$



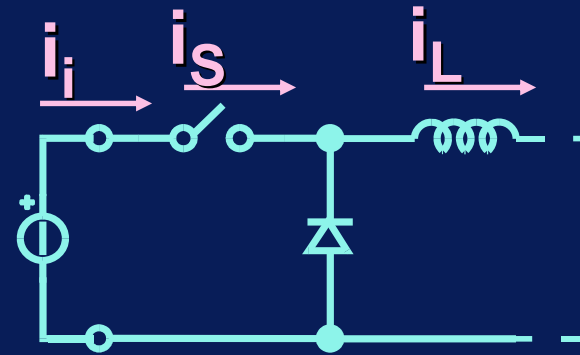
**Ciò è coerente con la conservazione della potenza:**

$$P_i = P_o \quad \Rightarrow \quad U_i \cdot I_i = U_o \cdot I_o$$

**Nota:**

$$I_i = I_s = I_L \cdot \delta = I_o \cdot \delta$$

$$P_i = U_i \cdot I_i = U_i \cdot I_o \cdot \delta$$



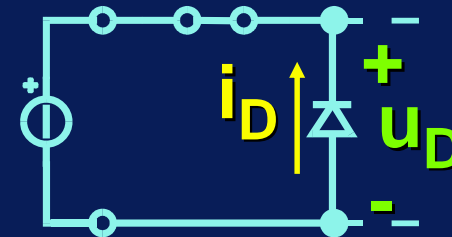
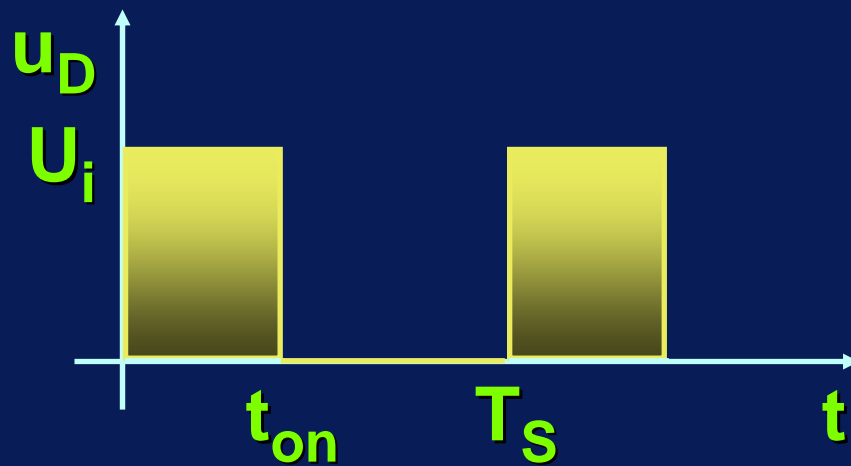
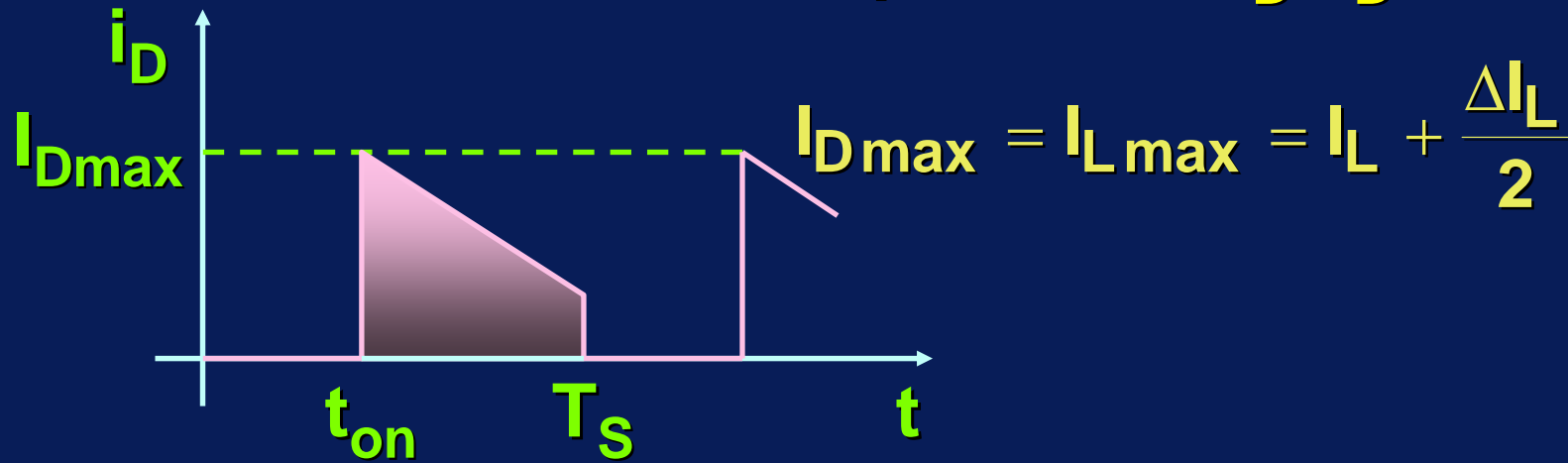
**Ciò è coerente con la conservazione della potenza:**

$$P_i = P_o \quad \Rightarrow \quad U_i \cdot I_i = U_o \cdot I_o$$

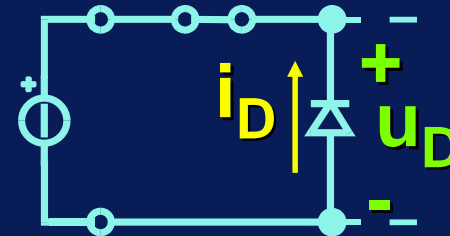
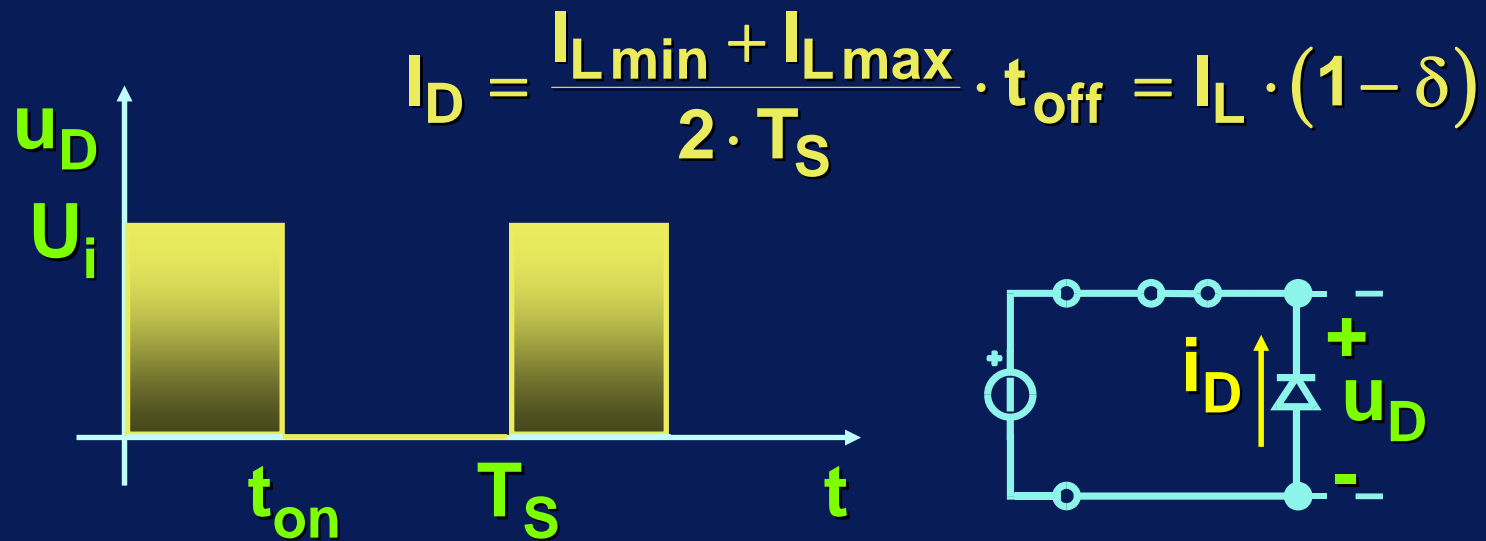
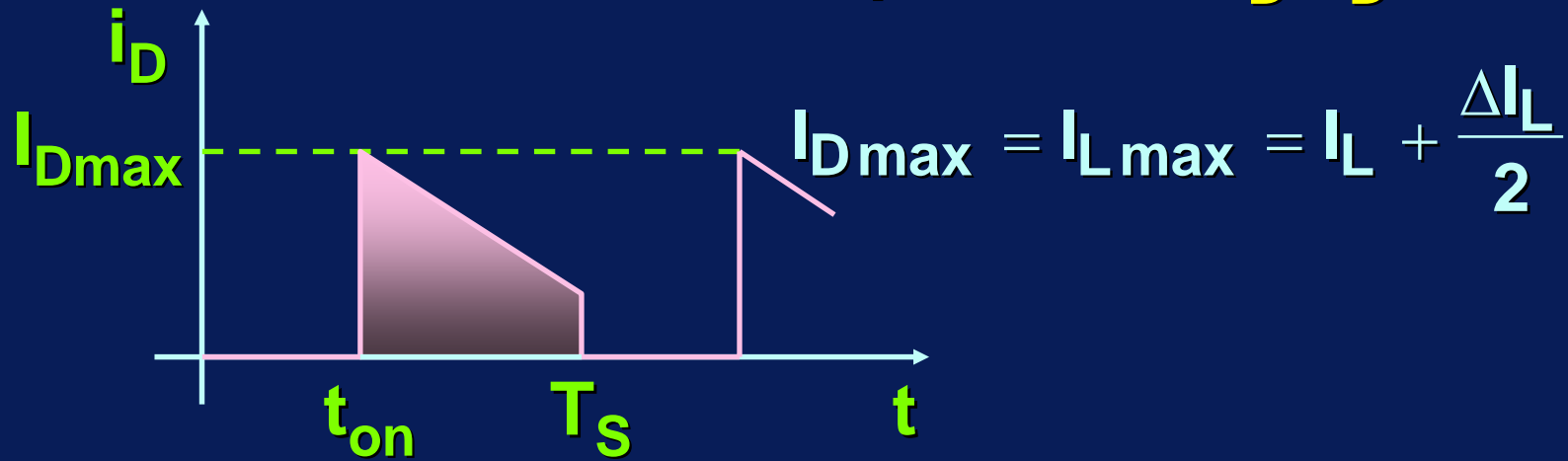
da cui:

$$\frac{I_i}{I_o} = \frac{U_o}{U_i} = \delta$$

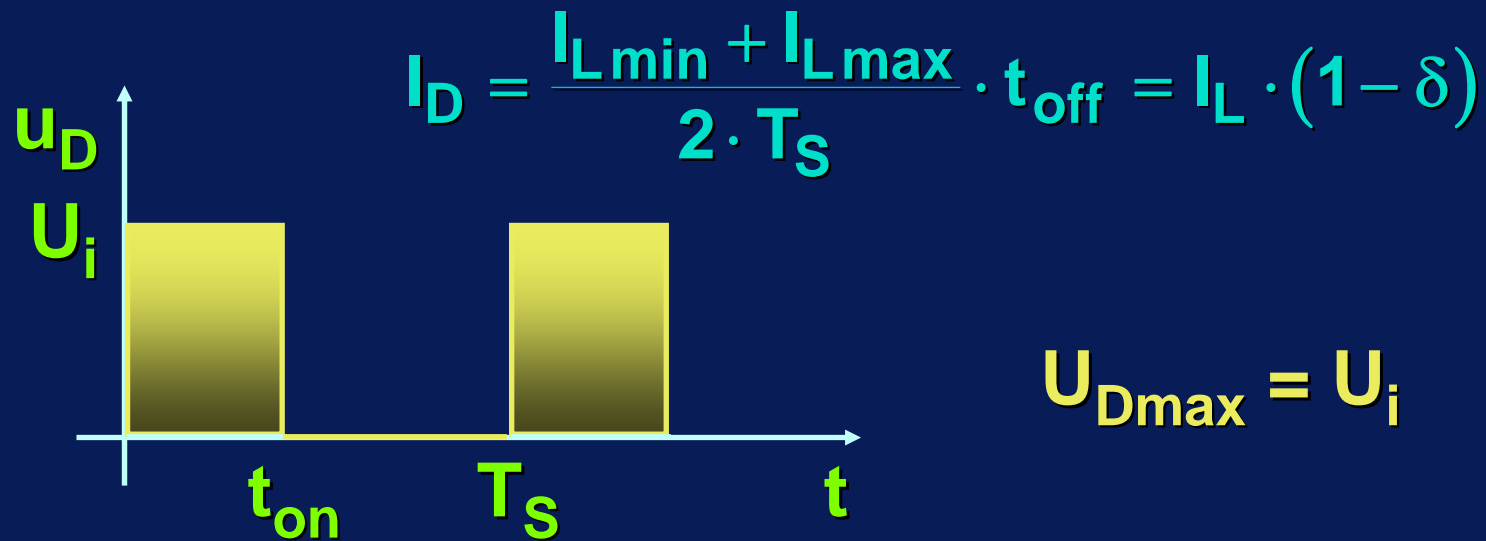
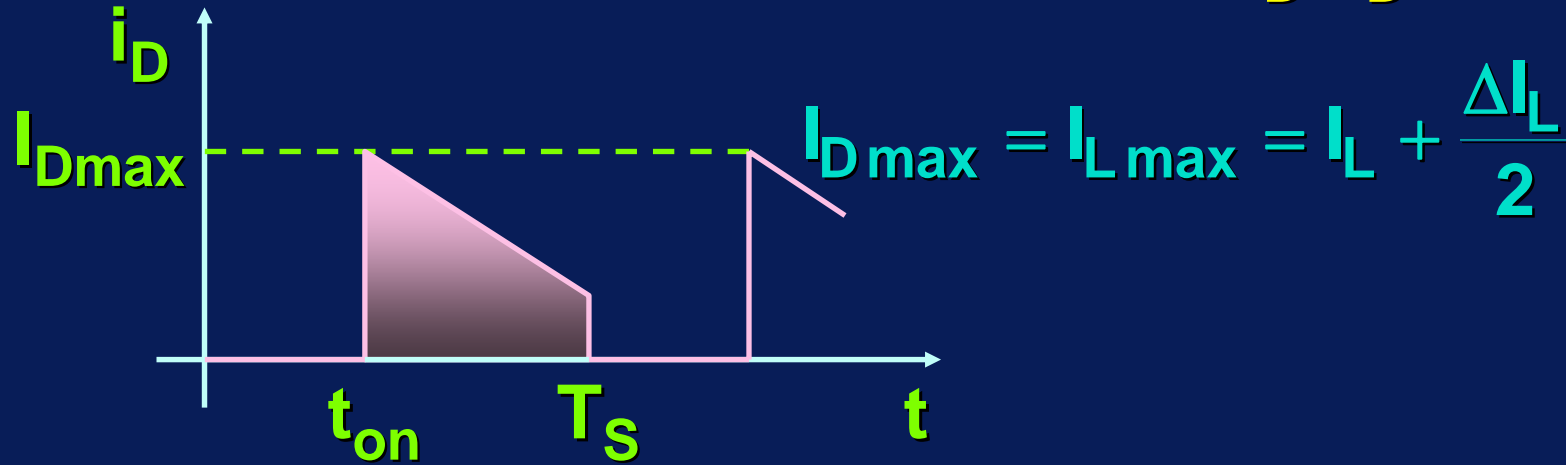
## Forme d'onda compressive: $u_D$ , $i_D$



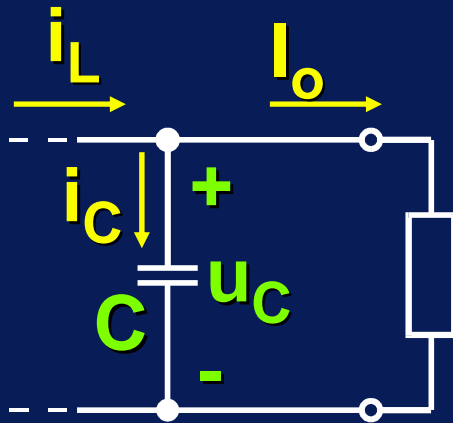
## Forme d'onda compressive: $u_D$ , $i_D$



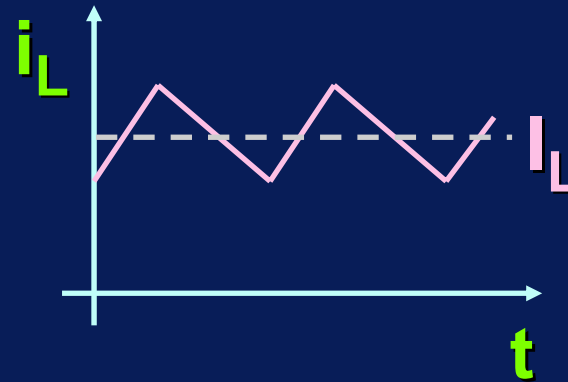
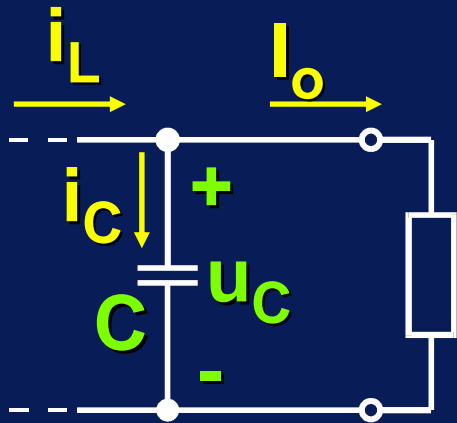
## Forme d'onda compressive: $u_D$ , $i_D$



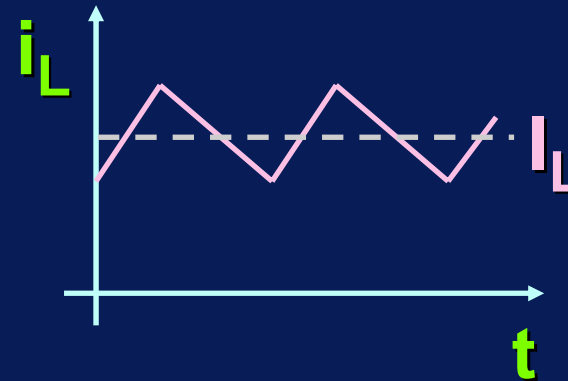
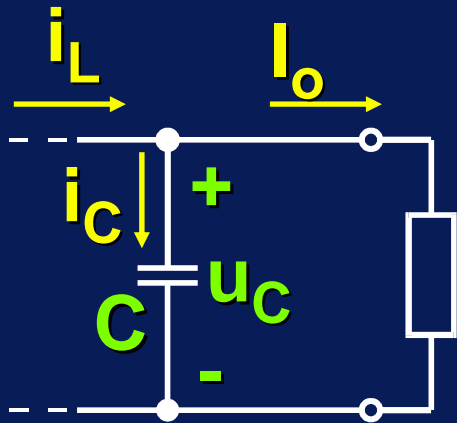
## Forme d'onda compressive: $i_C$



# Forme d'onda compressive: $i_C$



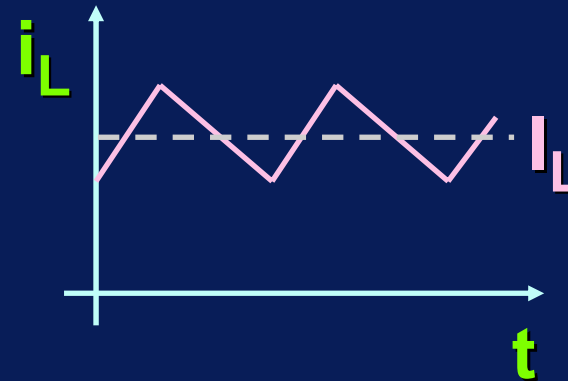
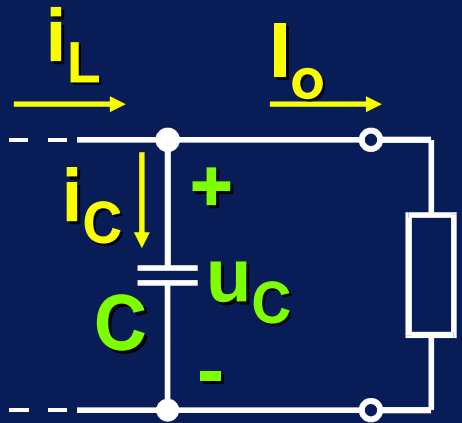
## Forme d'onda compressive: $i_C$



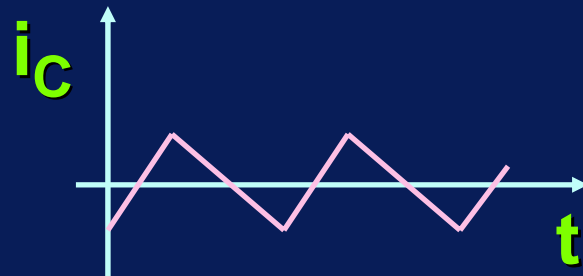
A regime:  $I_L = I_o \Rightarrow i_C = i_L - I_o$



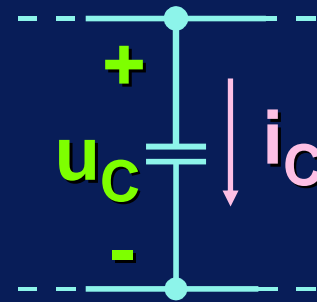
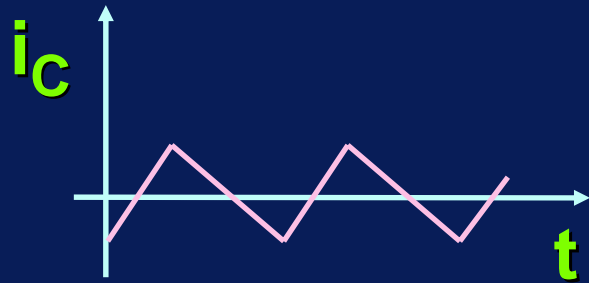
## Forme d'onda compressive: $i_C$



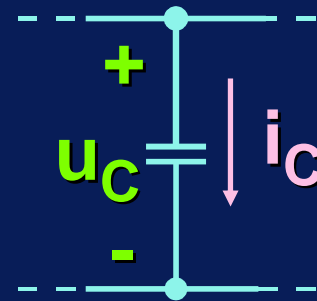
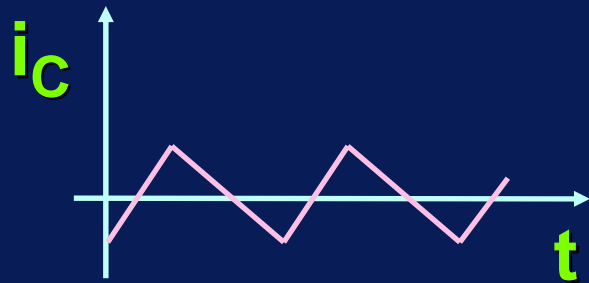
A regime:  $I_L = I_o \Rightarrow i_C = i_L - I_o$



# Forme d'onda compressive: $u_C$

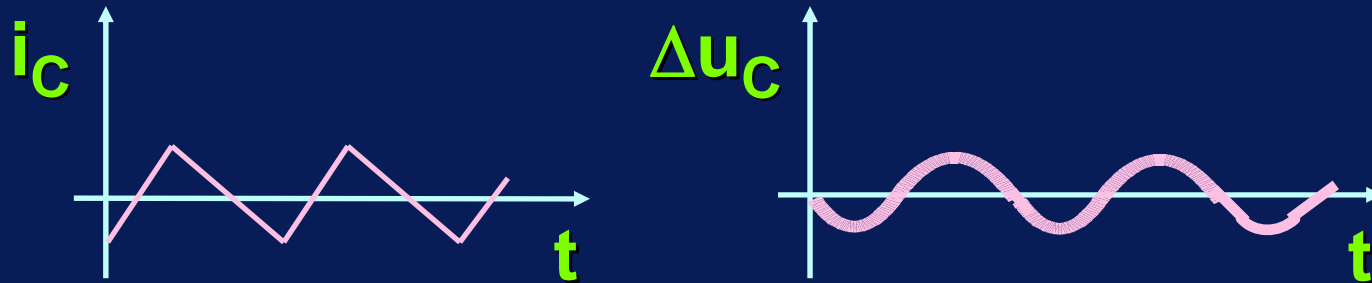


## Forme d'onda compressive: $u_C$



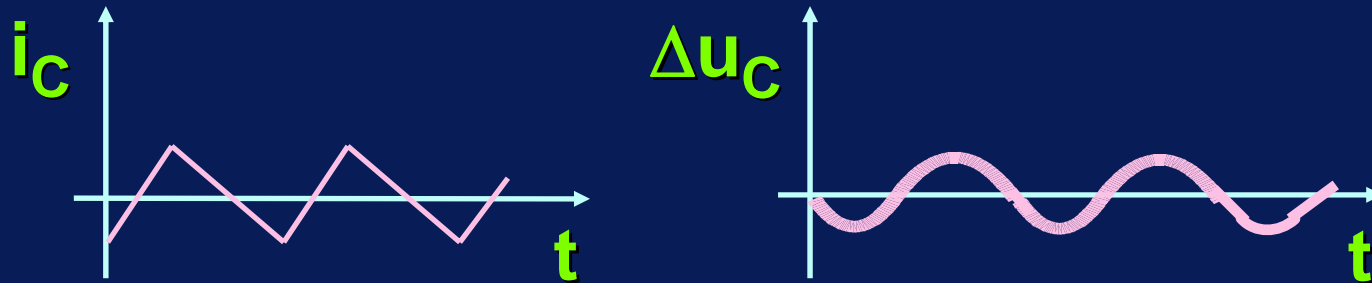
$$\Delta u_C = u_C - U_0 = \frac{1}{C} \cdot \int i_C dt$$

## Forme d'onda compressive: $u_C$



$$\Delta u_C = u_C - U_0 = \frac{1}{C} \cdot \int i_C dt$$

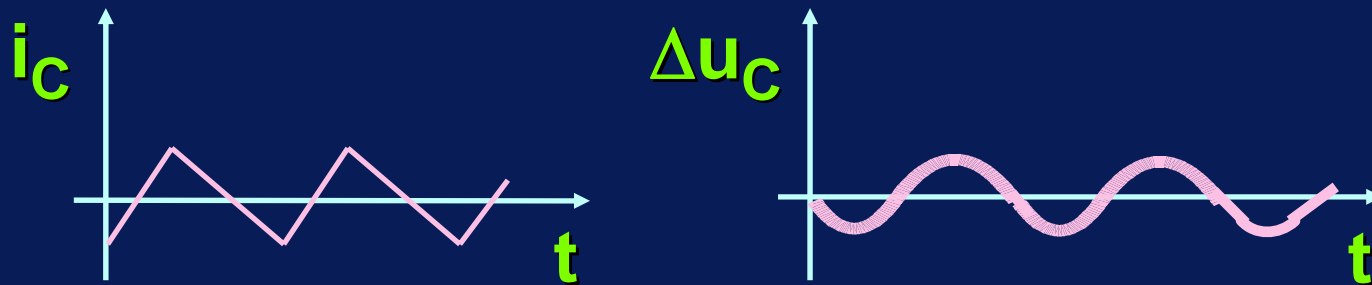
## Forme d'onda compressive: $u_C$



$$\Delta u_C = u_C - U_o = \frac{1}{C} \cdot \int i_C dt$$

$$\Delta u_C = \frac{1}{2 \cdot C} \cdot \left( I_{Cmax} \cdot \frac{t_{on} + t_{off}}{2} \right)$$

## Forme d'onda compressive: $u_C$



$$\Delta u_C = u_C - U_o = \frac{1}{C} \cdot \int i_C dt$$

$$\Delta u_C = \frac{1}{2 \cdot C} \cdot \left( I_{Cmax} \cdot \frac{t_{on} + t_{off}}{2} \right)$$
$$= \frac{1}{2 \cdot C} \cdot \frac{\Delta I_L}{2} \cdot \frac{T_S}{2} = \frac{\Delta I_L}{8 \cdot f_S \cdot C}$$

## Esempio di dimensionamento

$$U_i = 24V \pm 20\%$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 1\% U_o$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$



## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$\delta = \frac{U_o}{U_i}$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\delta = \frac{U_o}{U_i} = \frac{12}{U_{i \max}} \div \frac{12}{U_{i \min}}$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\delta = \frac{U_o}{U_i} = \frac{12}{U_{i\max}} \div \frac{12}{U_{i\min}} = 0.416 \div 0.625$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$U_o = 12V$$

$$I_o = 0 \div 2A$$

$$\Delta U_o \leq 120mV$$

$$\delta = \frac{U_o}{U_i} = \frac{12}{U_{i\max}} \div \frac{12}{U_{i\min}} = 0.416 \div 0.625$$

$$\Delta I_L = \frac{U_i - U_o}{L} \cdot t_{\text{on}} = \frac{U_o}{L} \cdot t_{\text{off}}$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$U_o = 12V$$

$$I_o = 0 \div 2A$$

$$\Delta U_o \leq 120mV$$

$$\delta = \frac{U_o}{U_i} = \frac{12}{U_{i\max}} \div \frac{12}{U_{i\min}} = 0.416 \div 0.625$$

$$\Delta I_L = \frac{U_i - U_o}{L} \cdot t_{\text{on}} = \frac{U_o}{L} \cdot t_{\text{off}} = \frac{U_o}{f_s L} \cdot (1 - \delta)$$

# Esempio di dimensionamento

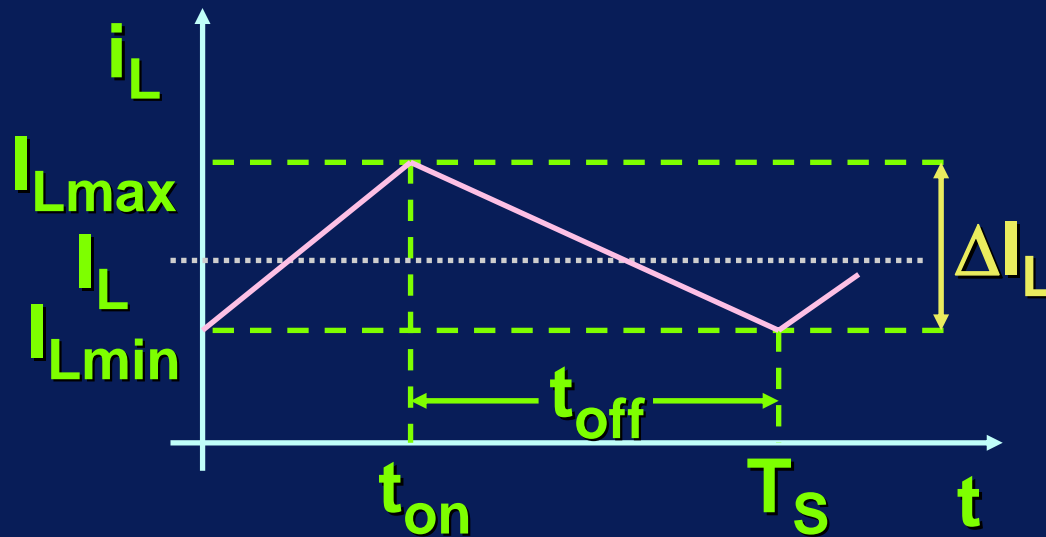
$$U_i = 19.6V \div 28.8V$$

$$U_o = 12V$$

$$I_o = 0 \div 2A$$

$$\Delta U_o \leq 120mV$$

$$\delta = \frac{U_o}{U_i} = \frac{12}{U_{i\max}} \div \frac{12}{U_{i\min}} = 0.416 \div 0.625$$



## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$U_o = 12V$$

$$I_o = 0 \div 2A$$

$$\Delta U_o \leq 120mV$$

$\Delta I_L$  è massimo quando  $\delta$  è minimo (se  $U_o$  è costante)

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$



## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

**Posto:**  $\Delta I_{Lmax} = 0.2 I_{on}$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$U_o = 12V$$

$$I_o = 0 \div 2A$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

**Posto:**  $\Delta I_{Lmax} = 0.2 I_{On}$  (CCM per  $I_o > 0.1 I_{On}$ )

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

**Posto:**  $\Delta I_{Lmax} = 0.2 I_o$  e  $f_s = 100kHz$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

**Posto:**  $\Delta I_{Lmax} = 0.2 I_o$  e  $f_s = 100kHz$

$$L = \frac{U_o}{f_s \Delta I_{Lmax}} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta I_L = \frac{U_o}{f_s L} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right)$$

**Posto:**  $\Delta I_{Lmax} = 0.2 I_o$  e  $f_s = 100kHz$

$$L = \frac{U_o}{f_s \Delta I_{Lmax}} \cdot \left( 1 - \frac{U_o}{U_{imax}} \right) = 175\mu H$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta U_{Cmax} = \frac{\Delta I_{Lmax}}{8f_s C}$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$\Delta U_{Cmax} = \frac{\Delta I_{Lmax}}{8f_s C} \quad \rightarrow \quad C = \frac{\Delta I_{Lmax}}{8f_s \Delta U_{Cmax}}$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$C = 4.16 \mu F$$



## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$C = 4.16 \mu F \quad \rightarrow \quad C = 4.7 \mu F$$

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

$$C = 4.16 \mu F \quad \rightarrow \quad C = 4.7 \mu F$$

**Pulsazione di risonanza:**

## Esempio di dimensionamento

$$U_i = 19.6V \div 28.8V$$

$$I_o = 0 \div 2A$$

$$U_o = 12V$$

$$\Delta U_o \leq 120mV$$

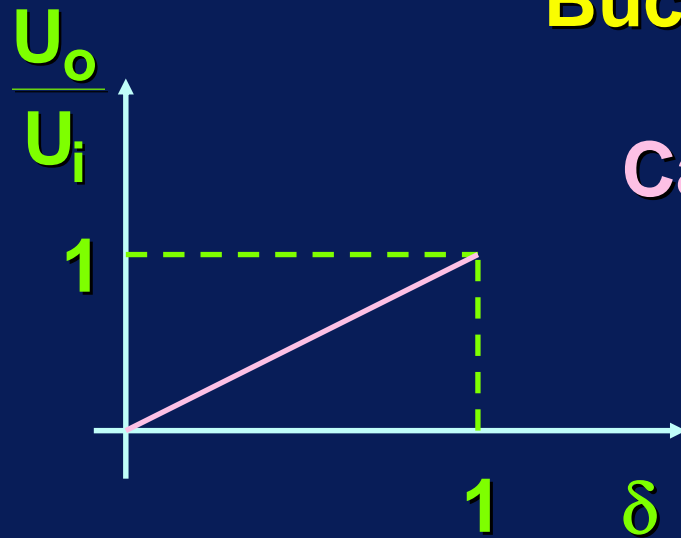
$$C = 4.16 \mu F \quad \rightarrow \quad C = 4.7 \mu F$$

**Pulsazione di risonanza:**

$$\omega_r = \frac{1}{\sqrt{LC}} \cong 35 \text{krad/s}$$

# **Caratteristiche statiche del convertitore Buck in CCM**

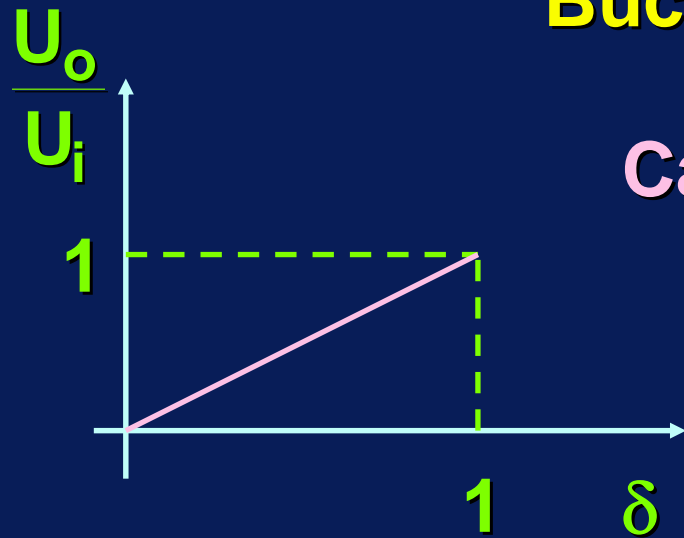
# Caratteristiche statiche del convertitore Buck in CCM



Caratteristica di controllo

$$U_o = \delta U_i$$

# Caratteristiche statiche del convertitore Buck in CCM



Caratteristica di controllo

$$U_o = \delta U_i$$

Controllo lineare

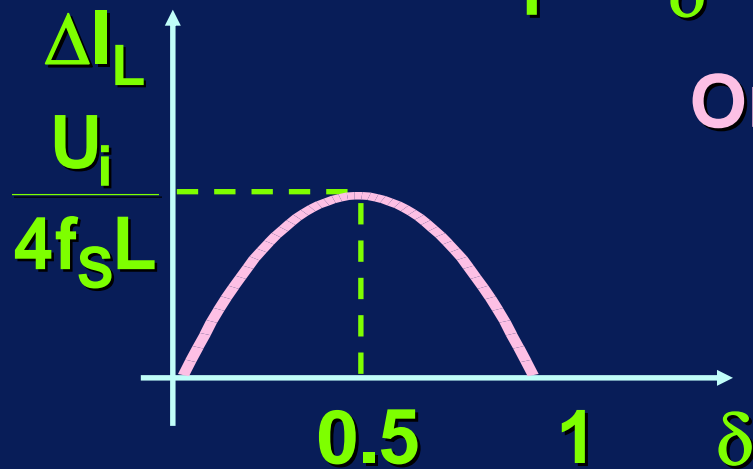
# Caratteristiche statiche del convertitore Buck in CCM



Caratteristica di controllo

$$U_o = \delta U_i$$

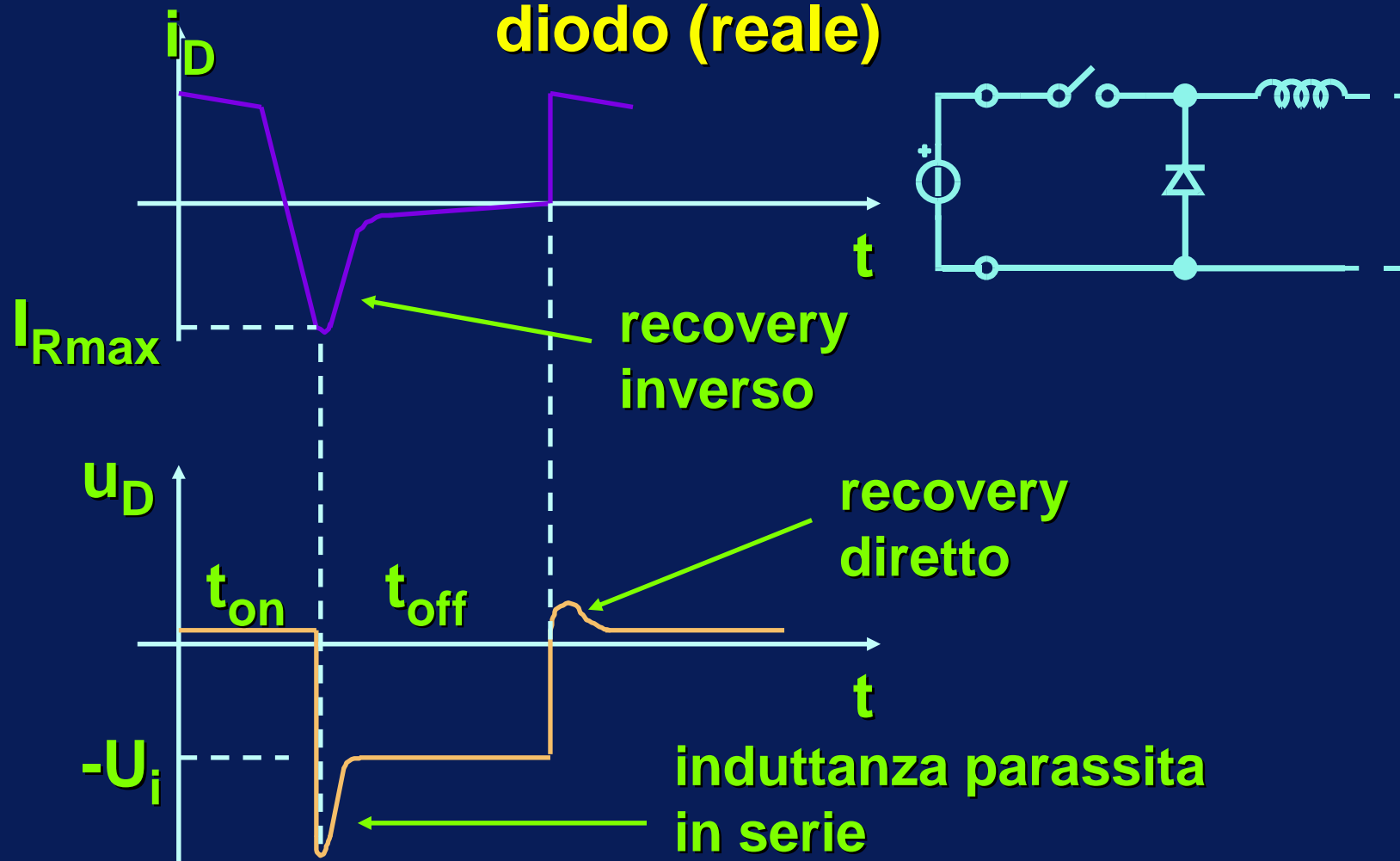
Controllo lineare



Ondulazione di corrente

$$\Delta I_L = \frac{U_i \cdot \delta \cdot (1 - \delta)}{f_s \cdot L}$$

# Andamenti di corrente e tensione del diodo (reale)

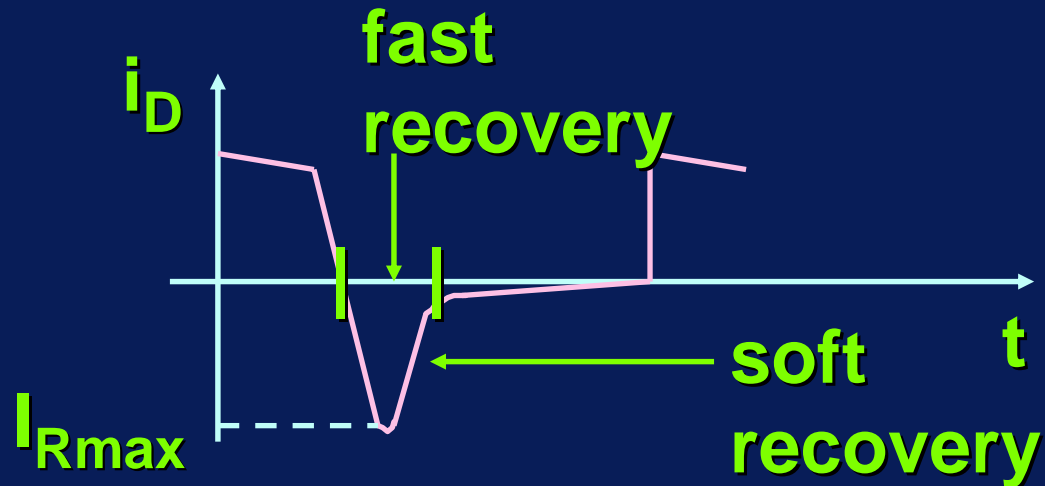




# Osservazioni

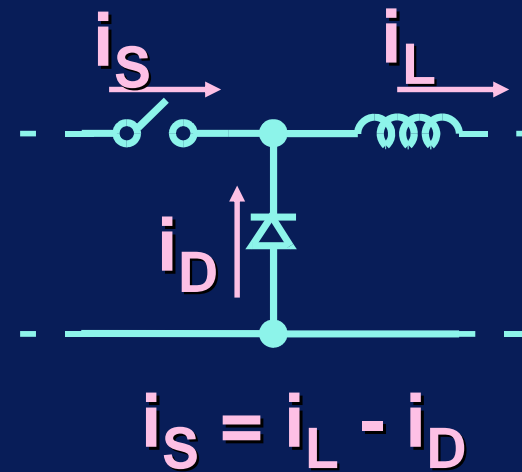
- La corrente di recovery inverso può avere lo stesso ordine di grandezza della corrente diretta
- Al recovery inverso sono associate perdite
- Il recovery diretto é normalmente ininfluente
- I diodi vanno scelti soft-recovery (per ridurre la sovratensione) e fast-recovery (per ridurre  $I_{Rmax}$  e le perdite)

# Osservazioni

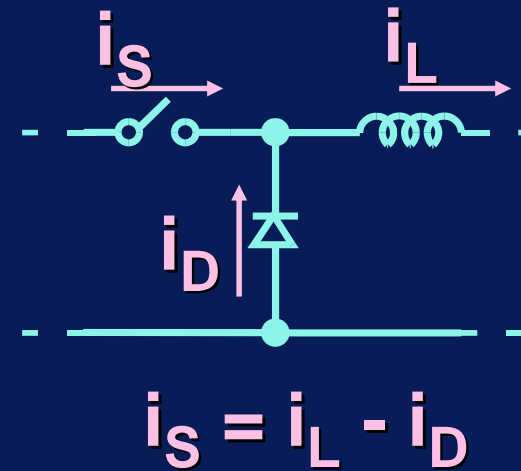
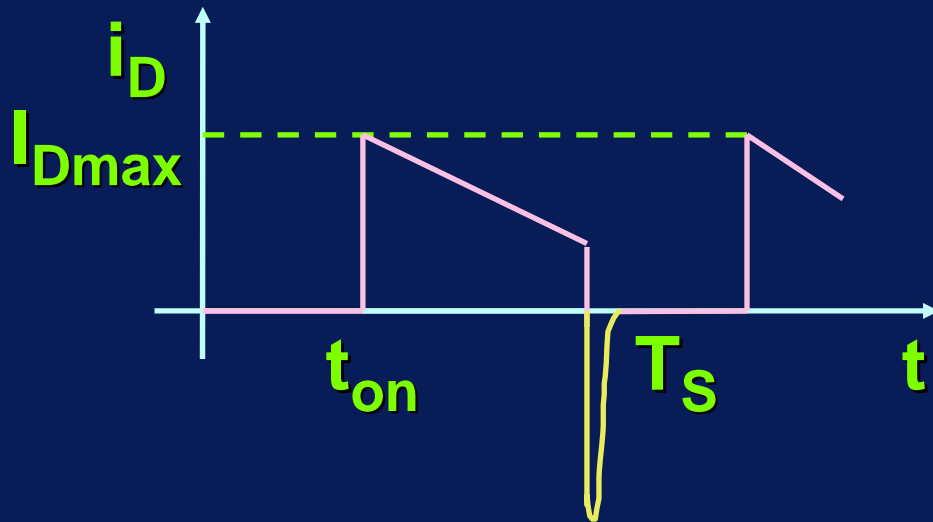


- I diodi vanno scelti soft-recovery (per ridurre la sovratensione) e fast-recovery (per ridurre  $I_{Rmax}$  e le perdite)

## Andamenti reali della corrente in S e D

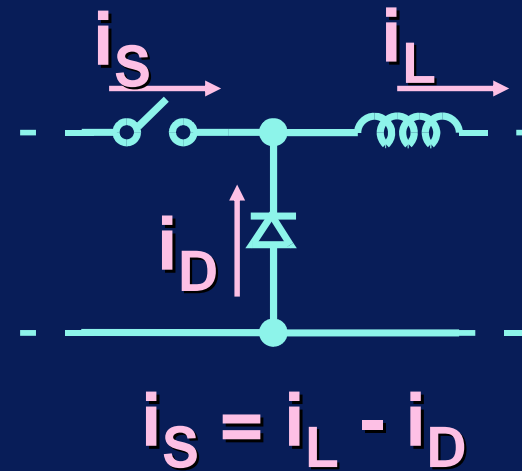
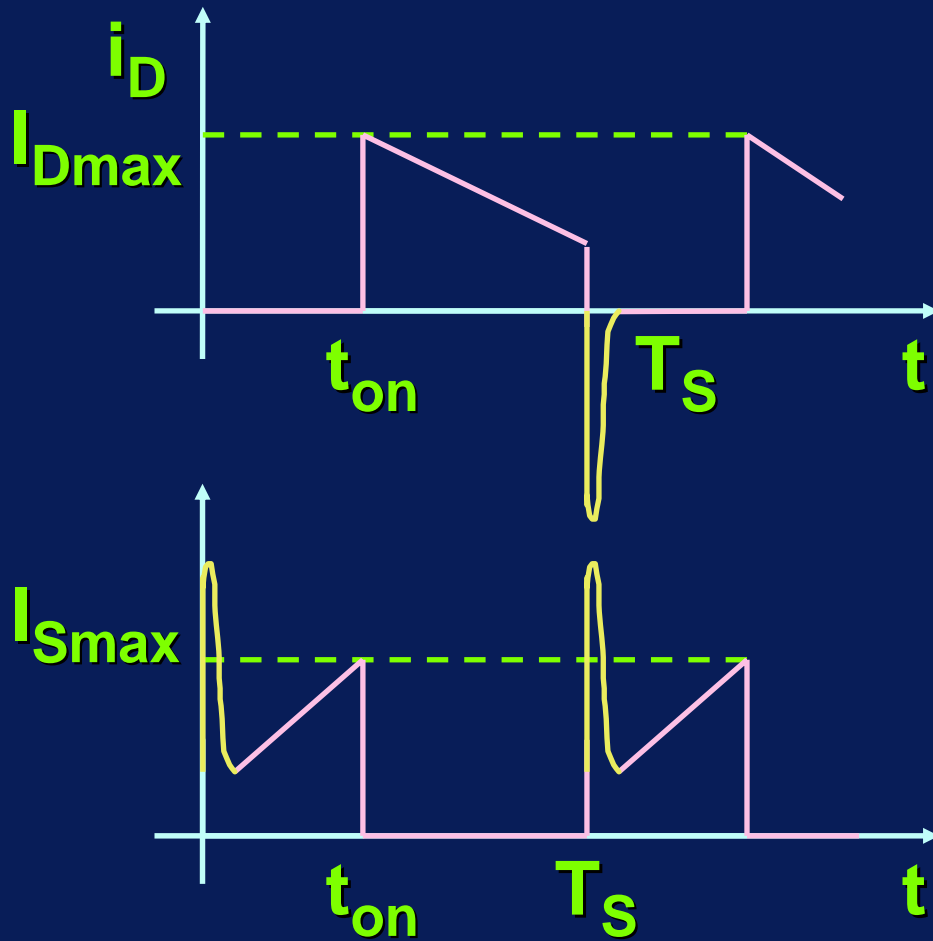


## Andamenti reali della corrente in S e D



**Il recovery inverso del diodo causa sovracorrenti nell'interruttore**

# Andamenti reali della corrente in S e D



# Conclusioni

- **Si è analizzato il funzionamento continuo (CCM) del convertitore abbassatore di tensione (buck)**
- **Il convertitore ha una caratteristica di controllo lineare**
- **I parametri del filtro vengono scelti per limitare l'ondulazione della tensione d'uscita e della corrente nell'induttanza**
- **Il filtro risulta tanto più piccolo quanto più elevata è la frequenza di commutazione**
- **Le sollecitazioni in tensione e corrente sono influenzate dal recovery inverso del diodo**