

<b>2015-03766</b>	<b>Enqvist, Per</b>	<b>NT-14</b>
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### Information about applicant

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**Project site:** Institutionen för Matematik

### Information about application

**Call name:** Forskningsbidrag Stora utlysningen 2015 (Naturvetenskap och teknikvetenskap)  
**Type of grant:** Projektbidrag  
**Focus:** Fri  
**Subject area:**

**Project title (english):** Identification of linear time-variant systems  
**Project start:** 2016-01-01 **Project end:** 2019-12-31  
**Review panel applied for:** NT-14, NT-1  
**Classification code:** 10106. Sannolikhets teori och statistik, 20205. Signalbehandling, 20202. Reglerteknik  
**Keywords:** Spektralestimering, Icke-stationära processer, Entropimaximering, Konvex optimering, Momentproblem

### Funds applied for

<b>Year:</b>	2016	2017	2018	2019
<b>Amount:</b>	859,557	912,597	970,938	1,033,305

## Descriptive data

### Project info

#### Project title (Swedish)\*

Identifiering av linjära tidsberoende system

#### Project title (English)\*

Identification of linear time-variant systems

#### Abstract (English)\*

In many applications a time-varying stochastic process is considered. For time-invariant processes there are plenty of methods for identifying and describing the process in both the time and frequency domain.

Most approaches to identification of time-varying processes are based on segmentation of the signal, where each segment is short enough to be considered as stationary, and then each of these segments are identified using one of the methods for time-invariant methods.

Here, we will develop methods for estimating dynamic models from features obtained from the evolutionary spectrum. The features will be obtained by applying time-variant input-to-state filters to calculate the time-variant equivalents of (time-invariant) input-to-state covariances. Given these features we will apply the method of moments based framework for generalized maximum entropy estimation developed by Lindquist, Bymes, Georgiou, et.al. The aim is to develop a complete estimation theory that can handle different model classes and feature sets and find robust and high resolution estimates with guaranteed stability, as in the stationary case.

## Popular scientific description (Swedish)\*

De flesta av de ljud, videor, väderobservationer, et.c., som vi omges av har en naturlig dynamik. Genom att betrakta korta tidsförlopp kan dessa modelleras med icke-dynamiska modeller som sedan kan sammanfogas för att t.ex. simulera motsvarande förlopp. Men ibland, som när man analyserar seismisk data och EEG är man intresserad av just ändringarna i dynamiken och det är lätt att missa dessa effekter om man utgår ifrån icke-dynamiska modeller. Därför vill vi kunna anpassa dynamiska modeller utifrån uppmätt data från en observerad process. Angreppssättet är att använda metodik som utvecklats för icke-dynamisk modellering och tillämpa här. Den bygger på att man mäter vissa karaktäristiska värden från signalen man betraktar och sedan tillpassar en modell som man antar har vissa önskade egenskaper. T.ex. vill man att modellen ska vara stabil, och man vill oftast att den ska ha ett reguljärt beteende, och detta åstadkoms genom att använda mått som entropi för att styra valet av modell. Mycket av arbetet kommer att ligga i att bestämma hur de karaktäristiska värdena ska utformas för att kunna estimeras från uppmätta signaler och ge bra modellenpassning, och i utformningen av måtten för att man ska kunna bevisa att man erhåller de önskade egenskaperna. Dessutom kommer algoritmer som använder regularisering och numeriskt effektiva och stabila tekniker, som krävs för optimeringen som bestämmer den tillpassade modellen, att behöva utvecklas.

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## Project period

### Number of project years\*

4

### Calculated project time\*

2016-01-01 - 2019-12-31

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## Classifications

Select a minimum of one and a maximum of three SCB-codes in order of priority.

Select the SCB-code in three levels and then click the lower plus-button to save your selection.

**SCB-codes\***

1. Naturvetenskap > 101. Matematik > 10106. Sannolikhets teori och statistik
2. Teknik > 202. Elektroteknik och elektronik > 20205. Signalbehandling
2. Teknik > 202. Elektroteknik och elektronik > 20202. Reglerteknik

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Enter a minimum of three, and up to five, short keywords that describe your project.

**Keyword 1\***

Spektralestimering

**Keyword 2\***

Icke-stationära processer

**Keyword 3\***

Entropimaximering

**Keyword 4**

Konvex optimering

**Keyword 5**

Momentproblem

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## Research plan

### Ethical considerations

Specify any ethical issues that the project (or equivalent) raises, and describe how they will be addressed in your research. Also indicate the specific considerations that might be relevant to your application.

#### Reporting of ethical considerations\*

Inga etiska frågor är aktuella för detta projekt

#### The project includes handling of personal data

No

#### The project includes animal experiments

No

#### Account of experiments on humans

No

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## Research plan

# PROPOSED RESEARCH PROJECT

## ”Identification of linear time-variant systems”

Per Enqvist

### Abstract

In many applications a time-varying stochastic process is considered. For time-invariant processes there are plenty of methods for identifying and describing the process in both the time and frequency domain. Most approaches to identification of time-varying processes are based on segmentation of the signal, where each segment is short enough to be considered as stationary, and then each of these segments are identified using one of the methods for time-invariant methods.

Here, we will develop methods for estimating dynamic models from features obtained from the evolutionary spectrum. The features will be obtained by applying time-variant input-to-state filters to calculate the time-variant equivalents of (time-invariant) input-to-state covariances. Given these features we will apply the method of moments based framework for generalized maximum entropy estimation developed by Lindquist, Byrnes, Georgiou, *et.al.*. The aim is to develop a complete estimation theory that can handle different model classes and feature sets and find robust and high resolution estimates with guaranteed stability, as in the stationary case.

## 1 Purpose and Aim

This research project aims at improving the methods used for estimation of the spectra of signals such as speech, EEG signals, seismic data, image data, econometric data *et.c.*. Common for these signals is that the dynamics changes over time. The aim of this project is to develop high resolution methods that can accurately follow the changes in the dynamics and determines stable modelling filters of some specified model class. Using the improved models and spectral estimates enables deeper analysis of the processes generating the signals, more accurate synthesis and more reliable predictions.

More precisely, time-varying input-to-state filters are defined to form a versatile tool for handling the features describing the evolutionary spectra in a concise and uniform way. We will consider the estimation of these features from measured data with special attention to if they are realizable or not. Assuming they are, a model in some model class will be selected using a criteria based on maximum entropy or minimizing the distance to some given prior model. The aim is to find a criterium that generate a well posed problem so that there is a unique model with no poles outside the unit circle that comply with it.

For the method to be practically applicable we will also consider two more aspects of the estimation problems. Since the features will be determined from finite data there will be estimation errors and a regularization should be considered to take the error variance into account. This will be accomplished by allowing an approximation error in the feature matching. For stationary processes, circulant covariances can be used to determine an approximative solution of the original problem that can be solved efficiently using FFT methods, and the aim is to generalize this approach.

## 2 Survey of the field

The basic problem considered here is the estimation of *evolutionary spectrum* and models from *semi-stationary processes* [34]. In practice, we base our analysis on some given data from the process. We can conceptually think of the data as the output signal  $y$  of a time-variant system  $W(t)$  that is fed by an input signal  $e$  which is an orthogonal process.

$$\text{input } \xrightarrow{e(t)} \boxed{W(t, z)} \xrightarrow{y(t)} \text{output}$$

Determining the transfer function  $W(t, z)$  can now be regarded as an inverse problem based on the data  $\{y(t)\}$ . Compared to the time-invariant case which is usually an overdetermined problem (few parameters in the model), the time variant case leads to an under-determined problem (each parameter can take different values at each time  $t$ ). Some assumption on the character of the signal has to be imposed in order to obtain the best solution and define in which sense it is best. Most common is to impose some restriction on how the model can change over time, for example, it can be desired to have as small and smooth changes over time as possible, or, to be a piecewise constant functions with as few jumps as possible. The classic approach introduced in [34, 35], is to assume that the model is slowly varying, which is defined as a bounded width of the Fourier transform of  $W(t, z)$  around zero frequency [34]. (*n.b.* zero frequency corresponds to a constant)

The local energy content over frequency of a semi-stationary process is varying over time, and the *evolutionary spectrum* [34] is used to describe this. If the process  $e(t)$  has a spectral representation  $e(t) = \int e^{it\omega} dZ(\omega)$ , where  $E[|dZ(\omega)|^2] = d\mu(\omega)$  then the evolutionary spectra of  $y(t)$  is

$$dH_t(\omega) = |W(t, z)|^2 d\mu(\omega).$$

Estimation of evolutionary spectra is studied in [34] for slowly varying processes with bounded width, and in [45] using wavelets.

*Wavelet analysis* was developed to capture both the time-dependence and frequency dependence of signals [7, 46]. A wavelet is a function of time with a certain frequency content. From the mother wavelet a basis of  $L^2$  is generated by dilations and translations. Wavelets are used for both compression, such as JPEG2000, and analysis of signals.

Most slowly varying processes are analyzed by segmenting the signals into windows that are short enough so that each of them can be considered as a snapshot of a stationary process and

modelled as such. The original signal is typically prefiltered with a window function that is designed to give emphasis to the central part of the window and minimize the influence of the edges of the window. Then each windowed signal is modelled independently using the theory for time-invariant, stationary, processes. Hence, the methods for time-invariant processes are an important basis for the time-variant processes and most methods are developed on the same basic principles. The advantages of this method are that the modelling can be done independently for each window, and therefore applied sequentially as new data arrive, and that well studied methods for time-invariant processes can be directly applied. A disadvantage is that the segmentation is usually not adapted to the natural transitions between different dynamics in the process. Entropy and wavelets can be used to find an adapted segmentation [6], but using this method you have already taken the first step towards using a time-variant basis.

We proceed by considering a number of approaches that have been considered for time-variant processes and then go back to consider the time-invariant case.

One approach is to use a set of *time-variant basis functions* and describe the model parameters as a linear combination of them. This was done for AR models in [29], applied on speech signals in [23, 38], and for ARMA models in [22]. A main consideration is which basis functions to choose, and how many, since the variance and numerical properties depend on this choice. A second order expansion was used in [35], an arbitrary order expansion in [29], Legendre polynomials in [24], prolate spherical coordinates in [22], the Fourier basis in [23] and wavelet bases in [47, 21]. It was noted in [23, 38] that the roots of the time-dependent AR models sometimes wanders outside the unit circle. Even if this does not imply instability in the time-variant case, it will cause spikes in simulations and these models are generally avoided.

Then there are so called *random coefficient models*. Kitagawa and Gersch [25] approach the problem of fitting a time-varying AR model by assuming that the coefficients in the model are changing with independent normally distributed increments at each time step and then apply Kalman filtering and *maximum likelihood estimation*.

Next we will describe some different approaches used in the field to estimate the model  $W$  when the process is stationary, *i.e.*, the shaping filter  $W$  is time-invariant.

In practice it is common to first estimate the *power spectral density* (PSD)  $\Phi(e^{i\theta}) = |W(e^{i\theta})|^2$ , which describes the distribution of power in the frequency representation of the signal, and then determine  $W$  by spectral factorization.

If one wants to estimate the PSD the Maximum Likelihood method is known to provide efficient estimates. As the data length increase the estimate will then tend to the true PSD in the conceptual situation described above. But it is also well known that it is necessary to solve a non-convex optimization problem to find the estimate and the convergence of the available optimization methods is a problem, in particular for short data sequences and when there is no exact generating model.

Another common approach to estimation problems is to use the *method of moments*. The idea is that if you want to approximate a function and you know some moments, *e.g.*, function values or values of the derivatives, then a reasonable choice is to find a function, in some function class, such that it matches the given moments, *i.e.*, it interpolates them. In our



case the function to be determined is the (PSD) and the moments to be matched are often chosen as the covariances of the process  $y$ . In general, these interpolation problems have infinitely many solutions and traditionally the Burg entropy [1] is used to single out one particular solution. This so called central solution assumes a minimum of prior information, and in particular it holds no information on the zeros of the PSD. Given prior information about filter zeros, a Bayesian method would incorporate this information in the conditional probabilities and likewise the Burg entropy has been modified to reflect this information and improve on the central solution [5, 19]. During the last fifteen years the development of the theory for covariance interpolation has progressed a lot and these methods can now deal with bi-tangential and matrix valued interpolation constraints as well as multivariable densities [17].

I have worked on several aspects of this approach and next some of the contributions that are expected to be relevant for this project are described.

ARMA models can be estimated using an extended maximum entropy problem [27, 9, 20], where in addition to using a window of covariances as moments, a window of cepstrum parameters is also given. In particular, cepstrum parameters are calculated as the Fourier coefficients of the logarithm of the spectrum and the first coefficients thus roughly describes the envelope of the spectral density [31]. The cepstrum data substitutes the zero information, and even if there is a close relationship between the methods, this change of prior information changes some of the fundamental properties. The use of cepstrum data is well established in speech processing, [30], but has been a bit overlooked in spectrum estimation.

When the estimation is performed on data from real applications there are usually no true  $W$ , the input  $e$  is not a white noise signal, and there are noisy measurements of the output signal  $y$ . Then it is important to consider robust estimation methods that can be adapted to different assumptions on how the data is generated and to analyze how the determined transfer function model approximates the real system. When the moments are estimated it makes sense to consider *approximative moment matching* [14]. Some approaches that deal with interpolation of spectral densities of uncertain parameters, such as [39, 33] and [4, 41], used *hard constraints* in the form of prespecified ellipsoidal, or interval, regions for the parameters that had to be met. *Soft constraints* on the interpolation parameters have also been considered by introducing some *regularization*. For the combined covariance and cepstrum matching problem there is often no exact solutions and then it is critical that a good approximative solution can be found and regularized approaches are given in [9, 11].

The state of the art methods of moments uses *input-to-state covariances* [16], which generalizes the covariances, can be used to solve Nevanlinna-Pick interpolation problems, and their practical importance is that they can be used to adapt the resolution of the method to fit the application [3]. An important problem in practice for applying the methods is to find good estimates of the input-to-state covariances from data that has the right structure and are non-negative. This problem was recently studied in [32, 26, 13, 48], and generalizes the problem to estimate structured covariances [2]. Alternatively, rough estimates of the input-to-state covariances can be used with the approximative interpolation approach, in which case the regularization of the solution is handled in the approximative interpolation step [11].

An important fact to note is that the optimization formulation of these moment problems are global, *i.e.*, they are posed for general PSD functions, and no structure is imposed on the

PSD. This structure is a consequence of the distance measure and thus the information theory description. Some different distances have been advocated by different research groups. It has been shown that by replacing the Kullback-Leibler distance in [19] with the Hellinger distance [15], Itakura-Saito pseudo-distance [43, 44, 12], or optimal smoothing criterium [18] another set of spectral densities are obtained, but many good properties are preserved. I have also shown that the approximative interpolation approaches can be used in these methods [10], and can improve their robustness.

Problems with *Circulant covariances* have been studied to model periodic and skew-periodic random signals [28, 36]. The spectral measure then have discrete mass points which leads to that most calculations can efficiently be performed using FFT. This approach can also be used to approximate the solution to non-periodic signals.

All of the methods just described have their roots in the maximum entropy solution. A generalization to evolutionary spectrum estimation of the maximum entropy method [40] and Burgs method [8] was derived twenty years ago but despite reporting good results did not receive any attention. With the modern framework available we will use this result as a starting point for this program.

### 3 Project description

The purpose of this project is to draw from the knowledge and framework developed for the time-invariant modelling that I have participated in the development of, as described above.

As argued in Section 2, for time-invariant systems, entropy based interpolation methods for modelling of ARMA models can be formulated as a well posed problem, and solved using convex optimization. Furthermore, stability was guaranteed and excellent resolution could be obtained for short data sequences.

Here we will introduce time-varying input-to-state filters, generalize the maximum entropy approach and develop efficient optimization solvers to numerically determine the time-varying models. We divide the project into four tasks described next.

#### A: Feature Extraction

The first step is in introducing the *time-varying input-to-state filters*,

$$G(z, t) = (zI - A(t))^{-1} B(t),$$

which will enable us to develop the theory for a large class of basis functions in a unified setting. This will also set the scene for applying the machinery developed for the time-invariant case. The time-varying input-to-state filters will be used to reduce the information in the evolutionary spectrum to a time-varying input-to-state covariance matrix function. We will think of this as a feature that describes the process and its changes over time. The dependence of time is what makes this more complicated than for the stationary case and the implications of this will be investigated, in particular the geometric properties of the map has to be analyzed. Coupled to this development is the practical estimation problem of determining the time-varying input-to-state covariances that define the interpolation constraints. Here the theory in [13, 48] should be possible to extended to the time-varying case.

#### B: Model realization

The next step is the generalization of the maximum entropy problem. First we have to

consider the entropy concept for evolutionary spectra and analyse how the spectral distances considered for stationary spectra [19, 15, 18, 43, 44, 12] should be generalized to retain the well-posedness of the optimization formulations. A starting point is the evolutionary maximum entropy method in [40]. It would be most important to show that the models determined by this method have poles inside the unit circle, as for the time-invariant case. Considering different *distance measures* will have a direct impact on the model structure, both in the time and frequency domain. We saw earlier that different distances for stationary process leads to different rational and non-rational modelling filters. Using different distances to measure the change in the shaping/modulating function over time is also expected to generate corresponding variations of solutions. Here it expected that transportation measures can play an important role since they often agree better with our intuition of what it means for two spectrums to be close to each other than, *e.g.*, the  $L^2$  norm.

### C: Regularization

*Approximative interpolation* is used in the time-invariant [9, 11, 10] case both to obtain smoother solutions and also to determine the best candidates when no exact interpolants are available. The need for this kind of *regularization* is expected to be even more important for this more complex estimation problem.

### D: Numerical optimization

In this task we consider how the optimization problems defined in task B can be solved, the optimization algorithms, the numerical properties of the basis functions generated by the time-varying input-to-state filter. Furthermore, we also consider the possibility to approximate these problems with a numerically more tractable class of problems. The approach using *circulant covariances* [28, 36] for stationary processes leads to a class of problems where many of the calculations can be performed using FFT. It will be investigated how this can be performed for the time-varying case, the aim would be to achieve computationally efficiency using FFT and fast wavelet computations.

I expect that the approximative interpolation, the input-to-state covariance estimation and the numerical implementation of the optimization solver are the key elements to take this project from a theoretical construction to a practical applicable tool.

A rough plan for the distribution of time on the different tasks is given in Table 1. The long time span denoted for numerical optimization can be explained by numerical testing that will be initiated early on, but will not be in focus until later.

## 4 Significance

In the field of spectral estimation, there are many examples where an increased resolution of the estimates can have significant, and even life saving, impact. However, it is not only the resolution that is important; robustness and stability of the estimated models are also of great importance, as well as the numerical computations efficiency. By addressing all of these issues we hope to make important contributions to the field and next we mention a few of these application areas.

Seismic ground motion can be described by oscillatory processes [42]. The evolutionary power spectrum can be estimated and time-varying models can be determined from it and be

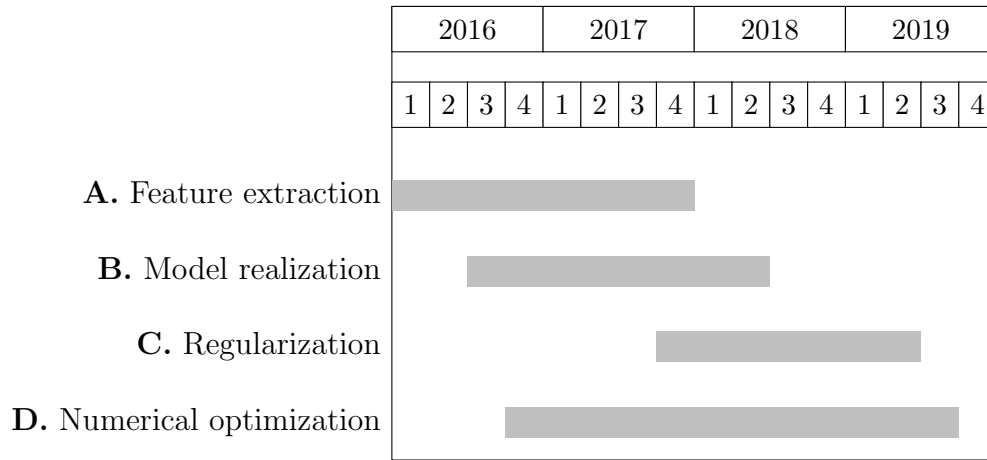


Table 1: *Time plan for the work.*

used for analysis and predictions. In turn, seismic waves cause damage to structures and can cause them to loose some of its functions and even collapse. Structured systems accumulate damage under service load and environmental excitations [21]. Dynamical models can describe the mode of operation, and changes in stiffness and damping, hence they are important to assess the condition of the system and diagnose failures.

Electroencephalography (EEG) signals [37, 47] are important for medical diagnosis. The dynamics of the signals changes very quickly. Fourier transforms and AR modelling have problems to capture these changes and therefore wavelets and time-varying AR modelling are increasingly popular.

In *speech and audio processing* the maximum entropy method of Burg is widely used. However, the duration of phonemes vary a lot between, *e.g.*, plosives and nasals, and therefore have a naturally time-dependent dynamic [22, 38].

In *image processing*, models are used for smoothing as well as increasing resolution of images. An image usually have several areas of different characteristics and if we describe the position of a pixel using the “time” index, this corresponds to time-varying dynamics. Using this technology, we can address magnetic resonance spectroscopy, ultrasound, telemedicine and similar important imaging applications.

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URL <http://www.sciencedirect.com/science/article/pii/S0005109812002300>

## Interdisciplinarity

### My application is interdisciplinary

An interdisciplinary research project is defined in this call for proposals as a project that can not be completed without knowledge, methods, terminology, data and researchers from more than one of the Swedish Research Councils subject areas; Medicine and health, Natural and engineering sciences, Humanities and social sciences and Educational sciences. If your research project is interdisciplinary according to this definition, you indicate and explain this here.

[Click here for more information](#)

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## Scientific report

### Scientific report/Account for scientific activities of previous project

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## Budget and research resources

### Project staff

Describe the staff that will be working in the project and the salary that is applied for in the project budget. Enter the full amount, not in thousands SEK.

Participating researchers that accept an invitation to participate in the application will be displayed automatically under Dedicated time for this project. Note that it will take a few minutes before the information is updated, and that it might be necessary for the project leader to close and reopen the form.

### Dedicated time for this project

Role in the project	Name	Percent of full time
1 Applicant	Per Enqvist	20
2 PhD Student	Doktorand	80

### Salaries including social fees

Role in the project	Name	Percent of salary	2016	2017	2018	2019	Total
1 Participating researcher	Doktorand	100	503,557	551,597	603,938	660,305	2,319,397
2 Applicant	Per Enqvist	20	250,000	250,000	250,000	250,000	1,000,000
Total			753,557	801,597	853,938	910,305	3,319,397

### Other costs

Describe the other project costs for which you apply from the Swedish Research Council. Enter the full amount, not in thousands SEK.

### Premises

Type of premises	2016	2017	2018	2019	Total
1 Offices	56,000	61,000	67,000	73,000	257,000
Total	56,000	61,000	67,000	73,000	257,000

### Running Costs

Running Cost	Description	2016	2017	2018	2019	Total
1 Conferences	Travel, hotel, conf.fee	50,000	50,000	50,000	50,000	200,000
Total		50,000	50,000	50,000	50,000	200,000

### Depreciation costs

Depreciation cost	Description	2016	2017	2018	2019
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### Total project cost

Below you can see a summary of the costs in your budget, which are the costs that you apply for from the Swedish Research Council. Indirect costs are entered separately into the table.

Under Other costs you can enter which costs, aside from the ones you apply for from the Swedish Research Council, that the project includes. Add the full amounts, not in thousands of SEK.

The subtotal plus indirect costs are the total per year that you apply for.

### Total budget

Specified costs	2016	2017	2018	2019	Total, applied	Other costs	Total cost
Salaries including social fees	753,557	801,597	853,938	910,305	3,319,397		3,319,397
Running costs	50,000	50,000	50,000	50,000	200,000		200,000
Depreciation costs					0		0
Premises	56,000	61,000	67,000	73,000	257,000		257,000
Subtotal	859,557	912,597	970,938	1,033,305	3,776,397	0	3,776,397
Indirect costs					0		0
Total project cost	859,557	912,597	970,938	1,033,305	3,776,397	0	3,776,397

### Explanation of the proposed budget

Briefly justify each proposed cost in the stated budget.

#### Explanation of the proposed budget\*

Budget avser

lön för en doktorand som anställs på 4 år och finansiering på 100%.

lön för handledning som utförs av projektledaren avseende 20% av lön.

lokalkostnader för doktoranden.

Resekostnader för presentation av resultat på konferenser. Första året räknar vi med en mindre summa för detta ändamål men att det kompenseras av inköp av nödvändig datorutrustning.

### Other funding

Describe your other project funding for the project period (applied for or granted) aside from that which you apply for from the Swedish Research Council. Write the whole sum, not thousands of SEK.

#### Other funding for this project

Funder	Applicant/project leader	Type of grant	Reg no or equiv.	2016	2017	2018	2019
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# CURRICULUM VITAE

Per Enqvist

## University degree

Civilingenjör (M.Sc. degree), Engineering Physics, Applied Mathematics, KTH, 1994.  
Diploma work supervisor: Prof. Clyde F. Martin, Texas Tech University, USA.  
Title of diploma work: “Control Theory and Splines; Applied to Signature Storage”

## Doctoral degree

Teknisk Doktor (Ph.D.) in Optimization and Systems theory, KTH, 2001.  
Title of thesis: “Spectral Estimation by Geometric, Topological and Optimization Methods”  
Supervisor: Anders Lindquist

## Post-doctorial positions

Time: April 2003 - Juli 2004

Place: Projet APICS

(formerly MIAOU)

INRIA

2004 Route des Lucioles - BP 93

FR-06902 Sophia Antipolis Cedex

France

Supervisor: Laurent Baratchart

Time: September 2001 - Mars 2003

Place: Istituto di Ingegneria Biomedica del CNR

(formerly LADSEB)

Area di Ricerca di Padova, Corso Stati Uniti, 4

I-35127 Padova

Italy

Supervisor: Andrea Gombani

## Docent title

Appointed 2011-11-02.

## Current position

*Lektor* (Associate professor)

Time: Permanent position since July, 2009.

Place: Division of Optimization and Systems theory  
Department of Mathematics  
KTH

## Former positions

*Forskarassistent* (Assistant professor)

Financed by Vetenskapsrådet - signals & systems.

Time: January, 2006 - June, 2009.

Place: Division of Optimization and Systems theory  
Department of Mathematics  
KTH

Teaching was performed in increasing fraction of the time, starting at 30% and finishing slightly over 50 %.

*Vikarierande Lektor* (Temporary Lecturer)

Time: July 2004 - July 2005

Place: Division of Optimization and Systems theory  
Department of Mathematics  
KTH

Full time teaching.

## Other merits

Project leader of ACCESS seed project on “Robust Spectral Estimation” within the ACCESS Linnaeus centre 2010-2011.

Member of ACCESS faculty.

Affiliated member of the Center for Industrial and Applied Mathematics (CIAM).

Responsible for the Systems track in the Aerospace master program.

Responsible for the Optimization and Systems theory seminar series.

I am currently main advisor of 1 industrial doctoral student (Göran Svensson)

I am currently coadvisor of 2 doctoral students (Johan Markdahl, Yuecheng Yang) (the main supervisor is Professor Xiaoming Hu).



# LIST OF PUBLICATIONS 2007-2015

Per Enqvist

Number of citations has been determined using Google scholar.

## Refereed publications in international journals

- J1** L. Baratchart, P. Enqvist, A. Gombani and M. Olivi, “Minimal symmetric Darlington synthesis”, Mathematics of Control, Signals, and Systems (MCSS), pages 283-311, Volume 19, Number 4 / November, 2007. ISSN 0932-4194 (Print) 1435-568X (Online).  
Number of citations: 2 (1)

The number in parenthesis indicate number of references by one of the authors.

## Articles in Proceedings

- P1** P. Enqvist, “Generalizing the Markov and covariance interpolation problem using input-to-state filters”, In Proceedings of the European Control Conference (ECC), Kos, Greece, 2007. ArXiv: 1104.1389.  
Number of citations: 0 (1)
- P2** P. Enqvist and E. Avventi, “Approximative covariance interpolation with a quadratic penalty”, Proceedings of the 46th IEEE Conference on Decision and Control (CDC), New Orleans, USA, 2007.  
Number of citations: 1 (4)
- P3** P. Enqvist and J. Karlsson, “Minimal Itakura-Saito distance and covariance interpolation”, In Proceedings of the 18:th Mathematical Theory of Networks and Systems (MTNS), July 28- Aug. 1, Virginia, USA, 2008.  
Number of citations: -
- P4** P. Enqvist and J. Karlsson, “Minimal Itakura-Saito distance and covariance interpolation”, 47th IEEE Conference on Decision and Control (CDC), Dec. 9-11, Cancun, Mexico, 2008.  
Number of citations: 12 (1)
- P5** (\*), P. Enqvist “Covariance Interpolation and Geometry of Power Spectral Densities”, Proceedings European Control Conference (ECC), Budapest, Hungary, 2009.  
Number of citations: 0

- P6** (\*) P. Enqvist “Approximative Covariance Interpolation”, 19:th Mathematical Theory of Networks and Systems (MTNS), July 5- July 9, Budapest, Hungary, 2010. arXiv:1104.1880.  
Number of citations: 0 (2)
- P7** Laurent Baratchart, Per Enqvist, Andrea Gombani and Martine Olivi, “Minimal Symmetric Darlington Synthesis: the Real Case.” In Proceedings of the 19:th Mathematical Theory of Networks and Systems (MTNS), Budapest, Hungary, 2010.  
Number of citations: 0 (1)
- P8** J Karlsson, P Enqvist, “Input-to-State Covariances for Spectral Analysis: The Biased Estimate.” In Proceedings of the 20:th Mathematical Theory of Networks and Systems (MTNS), Melbourne, Australia, 2012.  
Number of citations: 2 (0)
- P9** P Enqvist, “Spectral estimation from covariances and inverse-covariances” In Proceedings of the 21:th Mathematical Theory of Networks and Systems (MTNS), Groningen, Holland, 2014.  
Number of citations: 0

### Doctoral Thesis article

- DT1** (\*) P. Enqvist and E. Avventi, “Approximative Linear and Logarithmic Interpolation of Spectra”, In “Spectral Moment Problems: Generalizations, Implementation and Tuning”, Ph.D. Thesis of Enrico Avventi, Department of Mathematics, Royal Institute of Technology, Sweden, 2011. ISBN 978-91-7501-087-8.  
Number of citations: 3 (0)

### Technical Report

- TR1** P. Enqvist, and E. Avventi,  
“Approximative linear and logarithmic interpolation of spectra”,  
KTH Mathematics, TRITA-MAT 09 OS 02, ISSN 1401-2294, ISRN/KTH/OPT/R-09/02-SE, 2009.  
Number of citations: 0 (1)

### Own Patents

- Pat1** C.I. Byrnes, A. Lindquist, P. Enqvist, and S. Gusev, “Method and Apparatus for Speech Analysis and Synthesis”, United States Patent 5,940,791, August 17, 1999.



## Five most cited Papers

1. C.I. Byrnes, P. Enqvist, and A. Lindquist, “Cepstral coefficients, covariance lags and pole-zero models for finite data strings”, *IEEE Transactions on Signal Processing*, SP-50, April 2001.  
Number of citations: 38 (24)
2. (\*) P. Enqvist, “A homotopy approach to rational covariance extension with degree constraint”, *International Journal of Applied Mathematics and Computer Science*, vol. 11, no. 5, pp. 1173-1201, 2001.  
Number of citations: 40 (2)
3. C.I. Byrnes, P. Enqvist, and A. Lindquist, “Identifiability and well-posedness of shaping-filter parameterizations: a global analysis approach”, *SIAM Journal on Control and Optimization*, vol. 41, no. 1, pp. 23-59, 2002  
Number of citations: 21 (22)
4. P. Enqvist, “A convex optimization approach to ARMA(n,m) model design from covariance and cepstrum data”, *SIAM Journal on Control and Optimization*, vol. 43, no. 3, pp. 1011-1036, 2004.  
Number of citations: 12 (5)
5. P. Enqvist and J. Karlsson, “Minimal Itakura-Saito distance and covariance interpolation”, *47th IEEE Conference on Decision and Control (CDC)*, Dec. 9-11, Cancun, Mexico, 2008.  
Number of citations: 12 (1)



## CV

**Name:** Per Enqvist  
**Birthdate:** 19710922  
**Gender:** Male

**Doctorial degree:** 2001-04-06  
**Academic title:** Docent  
**Employer:** Kungliga Tekniska högskolan

## Research education

### Dissertation title (swe)

Spektralestimering med geometriska, topologiska och optimeringsmetoder

### Dissertation title (en)

Spectral Estimation by Geometric, Topological and Optimization Methods

### Organisation

Kungliga Tekniska Högskolan,  
Sweden  
Sweden - Higher education Institutes

### Unit

Institutionen för Matematik

### Supervisor

Anders Lindquist

### Subject doctors degree

10199. Annan matematik

### ISSN/ISBN-number

### Date doctoral exam

2001-04-06

## Publications

**Name:** Per Enqvist

**Birthdate:** 19710922

**Gender:** Male

**Doctorial degree:** 2001-04-06

**Academic title:** Docent

**Employer:** Kungliga Tekniska högskolan

Enqvist, Per has not added any publications to the application.

## Register

### Terms and conditions

The application must be signed by the applicant as well as the authorised representative of the administrating organisation. The representative is normally the department head of the institution where the research is to be conducted, but may in some instances be e.g. the vice-chancellor. This is specified in the call for proposals.

The signature *from the applicant* confirms that:

- the information in the application is correct and according to the instructions from the Swedish Research Council
- any additional professional activities or commercial ties have been reported to the administrating organisation, and that no conflicts have arisen that would conflict with good research practice
- that the necessary permits and approvals are in place at the start of the project e.g. regarding ethical review.

The signature *from the administrating organisation* confirms that:

- the research, employment and equipment indicated will be accommodated in the institution during the time, and to the extent, described in the application
- the institution approves the cost-estimate in the application
- the research is conducted according to Swedish legislation.

The above-mentioned points must have been discussed between the parties before the representative of the administrating organisation approves and signs the application.

*Project out lines are not signed by the administrating organisation. The administrating organisation only sign the application if the project outline is accepted for step two.*

*Applications with an organisation as applicant is automatically signed when the application is registered.*

