A Framework for Scheduling and Resource Allocation in LTE Downlink Using Nash Bargaining Theory

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***Abstract*—In this paper, we propose a game theoretical model for joint scheduling and radio resource allocation in the downlink of a Long Term Evolution system, where Orthogonal Frequency Division Multiple Access is used as the multiple access scheme. The context is that of *spectrum sharing*, with multiple users competing for the simultaneous access to the radio channel. We first give a layered system representation and then model it through a game theoretic formulation using Nash Bargaining theory, where players cooperate to achieve a better common payoff. A trade-off between fairness and throughput is identified and addressed. In addition, we also propose an efficient algorithm that drives the system toward a *balanced Pareto efficient* operating point represented by the Nash Bargaining Solution. Numerical results are also provided to show the validity of the proposed approach.**

***Index Terms*—Resource allocation, OFDMA, LTE downlink, Nash Bargaining Solution.**

I. INTRODUCTION

IN MODERN telecommunication systems, efficient spectrum usage is a key factor. Wireless technologies have been experiencing an ever increasing diffusion. A careful use of resources (i.e., time, frequency, power, etc.) is imposed by their scarcity with respect to the demand. Orthogonal Frequency Division Multiple Access (OFDMA) is a promising multiple-access technique for high data rate transmission, due to its capability of exploiting the multiuser diversity by adaptive resource allocation. This means that packet scheduling and physical layer resource assignment are dynamically performed according to the channel conditions and Quality of Service (QoS) experienced by the users.

An emerging scenario where this access scheme is employed is represented by the Long Term Evolution (LTE) of 3G systems [1], where OFDMA is used in the downlink. Here the available bandwidth is divided into a certain number of orthogonal bundles of subcarriers, called subchannels, and each of them is assigned to a certain user for a subframe period (1 ms).

If the resource allocation is driven only by the efficiency goal, then this may result in an unfair allocation from the point of view of the system users because those experiencing better conditions (e.g., closer to the base station or with a better channel quality) are given more service. On the other hand, if the only objective were the fair allocation of resources to users, then this could lead to an inefficient result due to a limited exploitation of users' different conditions. Therefore, harmonizing the two objectives is an important and non-trivial task.

In this paper, along the lines of [2], [3], we utilize a modular representation of the radio resource management procedure which is split between two functional entities, i.e., a credit-based scheduler and the actual resource allocator. The scheduler selects the candidate packets for the transmission among its internal queues. The Radio Resource Allocator (RRA) matches the packets to the allocation units of the LTE downlink [4]. Denote with D the number of packets proposed by the scheduler, and with L the number of those selected by the RRA. D is supposed to be greater than or equal to L. Here the trade-off among the two entities arises: when D is high, then the RRA is free to choose the subset of packets addressed to the users with better channel conditions, thus maximizing the overall throughput without considering fairness. On the other hand, when it is minimized then all the proposed packets are mandatorily allocated with no freedom of choice for the allocator, thus maximizing the fairness.

We model this situation by using the cooperative game theory framework, in particular the Nash Bargaining theory. The two entities are the two players of the game and they bargain on the number of packets to select with the aim of maximizing a common utility function, the Nash function. The theory guarantees the existence and uniqueness of a maximum point (Nash Bargaining Solution), which is a *Pareto efficient* solution having also the merit to strike a good balance between the objectives of the two players, i.e., guaranteeing equity at the end of the bargaining process. We provide an efficient yet effective procedure to reach that point as well. Some simple numerical evaluations in a realistic LTE system are shown to confirm the soundness of our analysis.

The structure of the paper is the following. Section II gives a review of the literature. In Section III our layered system model is discussed. Section IV introduces some basic definitions of the Nash Bargaining theory, while in Section V the problem is formulated using these concepts and our proposal is described. Supporting numerical results are shown in Section VI and conclusions are drawn in Section VII.

II. STATE OF THE ART

In the literature, many studies have been conducted to tackle the problem of resource allocation in OFDMA cellular networks, for both the uplink and the downlink. Most of them

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assume perfect knowledge, at the base station, of instantaneous Channel State Information (CSI), so as to exploit multiuser diversity to increase the efficiency. Several formulations of the problem exist and different mathematical tools have been used, also stressing different aspects.

A first powerful tool is constrained optimization, using, e.g., weighted sum rate maximization as the objective. For any fixed subchannel assignment, the optimum is reached by multilevel waterfilling [5] for the continuous rate relaxation, or by greedy and bisection allocation [6] for the discrete case. However, the sum rate maximization may not result in a fair allocation, especially for non-symmetric channels and non-uniform traffic patterns [7]. Therefore, some studies tried to consider a joint solution for an efficient yet fair allocation [7]–[9]. In general, exact optimization approaches suffer from the issue that the optimal subchannel assignment is a combinatorial problem whose complexity increases exponentially with the number of subcarriers. Moreover, typically the computation of an optimal solution is centralized and requires complete knowledge of the network. In [5], an efficient suboptimal algorithm is found considering a convex relaxation. In [10] a solution is found by using Lagrangian dual decomposition and considering that the duality gap goes to zero when the number of subcarriers tends to infinity.

Another way to approach the problem of resource allocation is through game theory. Terminals requesting access to the shared resources are seen as players of a game who compete in order to maximize their own utility, e.g., their data rate. In this way, the efficiency and development of the game are analyzed together with schemes enforcing players to move towards an efficient operating point. An overview of spectrum sharing games is given in [11] and [12]. Many alternatives are described, from the simple non-cooperative approach to more sophisticated bargaining and auction-based games. From a practical point of view, game theory is also seen as a way to derive efficient distributed algorithms for dynamic spectrum sharing with agents having only local information. Such solutions are easier to implement than a centralized one needing complete knowledge.

As examples closely related to the present paper, we mention [13], where a second-price auction mechanism is proposed to model user competition in a wireless fading channel, and bids are posed based on the perceived channel quality. The existence of a Nash equilibrium is proved, together with its Pareto optimality. Nash Bargaining Solutions (NBS) and coalitions are employed in [14] to formulate a problem of fair rate maximization and find a suboptimal distributed algorithm for uplink access. NBS and coalitions are also used in [15] for the case of OFDMA-based relay networks. The authors face both the problem of spectrum and power allocation among relay nodes within the same coalition, and then the inter-coalition coordination, and propose some greedy algorithms able to enhance the total system capacity and maintain the user fairness. In [3], the same system model of this work has been described and analysed under a non cooperative approach.

In the present paper, we adopt a problem formulation applying Nash Bargaining theory. The goal is not only to provide an original view of the system, but also to exploit the results of the game theoretic analysis for the derivation of an efficient algorithm to dynamically estimate the optimal value of an operating parameter.

III. System Model

We consider the downlink of an LTE system, that uses OFDMA multiplexing. We can adopt a two-tier view of the system (along the same lines of [2], [3]). At the top level we have a *packet scheduler*, which receives packet flows from the upper layers and selects the candidates for transmission according to an internal scheduling policy. At the bottom level we have a *radio resource allocator* (RRA), which matches each packet to be transmitted with one of the available allocation units according to an efficiency maximization objective. This two modules are coupled through a list of candidate packets for transmission; the scheduler fills it in and the allocator selects for transmission only the subset of packets whose destinations have a better channel quality, thus exploiting the multiuser diversity.

In the following, we call L the number of resource units that the resource allocator is entitled to assign. This is subject to a constraint $L \leq L_{\max}$, where $L_{\max}$ is the maximum value corresponding to assigning all of them. The value assigned to L is communicated to the scheduler by the resource allocator. Actually, this represents a loose form of cross-layer interaction among the modules, maintaining the modularity and tunability of the approach.

Given L, the scheduler determines the number D of packets to be sent to the resource allocator, with $D \geq L$. The exact choice of D influences the entire allocation. If $D = L$, the resource allocator has no degree of freedom as to which packets to allocate (it can only assign each packet to its best channel to the receiver). By increasing D, the resource allocator can achieve a higher throughput by selecting only L packets out of D, according to a channel-aware policy, although at the price of a possibly decreased fairness (users experiencing good channel conditions are favored).

Through a process of abstraction, we can see the system scheduler-RRA as a game between two players with contrasting utility functions. A bargaining process is repeated every time some packets must be selected. The goal is the achievement of a balance between the two interests, with a solution that does not favor any of the players. According to this view, we can think of modelling this situation by using the Nash Bargaining theory, whose solution represents a Pareto efficient *fair* point, where in this definition *fair* refers to an equal distribution of the payoffs between the scheduler and the RRA. In the following, we show how to apply this framework to our case.

IV. The Nash Bargaining Problem

Let $\mathcal{N} = \{1, ..., n\}$ denote the set of players of the game, and $\mathcal{S}$ denote a closed and convex set of $\mathbb{R}^n$ representing the set of all feasible payoffs that the players can get if they work together. We also assume that no agreement is reached, that is, the players do not cooperate, they get a payoff collectively denoted by $\mathbf{d} = (d_1, ..., d_n) \in \mathcal{S}$, which is called *disagreement point*. Suppose that the set $\{\mathbf{y} \in \mathcal{S} | y_i \geq d_i, \forall i \in \mathcal{N}\}$ is nonempty and bounded. Then, the pair $(\mathcal{S}, \mathbf{d})$ is called an *n-person bargaining problem*.

Within set $\mathcal{S}$, we use the Pareto optimality as a selection criterion for the bargaining solutions.

Definition 1: A point $\mathbf{p} = (p_1, ..., p_n)$ is said to be **Pareto optimal** if and only if there is no other solution $\mathbf{s} = (s_1, ..., s_n)$ of the game such that $s_i \geq p_i, \forall i$.

The number of Pareto optimal points might be infinite. Among all of them, the Nash Bargaining Solution provides a unique result under the following conditions, which represent the characteristics that a solution is supposed to satisfy in Nash's theory, and are thus considered as axioms.

Definition 2: A specific solution to the bargaining problem $(\mathcal{S}, \mathbf{d})$, denoted as $\phi(\mathcal{S}, \mathbf{d})$, is called a **Nash Bargaining Solution (NBS)**, if the following axioms are satisfied.

1) Weak Pareto Efficiency: there is no other vector $\mathbf{y} \in \mathcal{S}$ such that $\forall i \in \mathcal{N}, y_i > \phi_i(\mathcal{S}, \mathbf{d})$.
2) Individual Rationality: $\phi(\mathcal{S}, \mathbf{d}) \succeq \mathbf{d}$ ($\geq$ element-wise).
3) Invariance: For any affine transformation ψ of $\mathcal{S}$ onto itself, $\psi(\phi(\mathcal{S}, \mathbf{d})) = \phi(\psi(\mathcal{S}), \psi(\mathbf{d}))$.
4) Independence of Irrelevant Alternatives: For any closed convex set $\mathcal{G} \subseteq \mathcal{S}$, if $\phi(\mathcal{G}, \mathbf{d}) \in \mathcal{G}$, then $\phi(\mathcal{G}, \mathbf{d}) = \phi(\mathcal{S}, \mathbf{d})$.
5) Symmetry: If $\mathcal{S}$ is invariant under all exchanges of players, then $\forall i, j \in \mathcal{N}, \phi_i(\mathcal{S}, \mathbf{d}) = \phi_j(\mathcal{S}, \mathbf{d})$.

Given the above axioms, there is only one NBS satisfying them, as stated in the following theorem [16].

*Theorem 1 (**Existence and Uniqueness of NBS**):* There exists a unique solution to the bargaining problem that satisfies all the axioms in *Definition 2*, given by

$$\phi(\mathcal{S}, \mathbf{d}) = \underset{\mathbf{s} \in \mathcal{S}, s_i \geq d_i \forall i}{\operatorname{argmax}} \prod_{i=1}^{n} (s_i - d_i) \tag{1}$$

Following these theoretical notions, the cooperative game in a multiplayer system can be defined as follows. Every player has its own payoff function, which is upper bounded and has a nonempty, closed and convex support. The goal is to maximize all these functions at the same time. The problem addressed is to find a way to choose an operating point in $\mathcal{S}$ which is optimal and fair (i.e., not good only for some players). The NBS is a solution to this issue.

V. Problem formulation and proposed solution

In the system under investigation, the choice of D determines a trade-off between the possible objectives of throughput and fairness. For simplicity in the exposition, in the following analysis we consider a network scenario with only two destination users (referring to the receivers of packet flows, not the two entities the resource manager is split into), in addition to a base station. However, our results can be naturally extended to the general case with $M > 2$ users. The scheduler and the RRA can be considered as the two players of a game. Their action space is the set of values of D they can propose, i.e., $A_1 = A_2 = \{L, L+1, ..., 2L\}$ (up to ML in general). When the actions proposed, s_1 and s_2 coincide, then the payoffs are assigned as the fairness F for the scheduler and the throughput T for the RRA. The fairness F, measured using Jain's index [17], is a decreasing function of D and varies between 1 and 1/2 (or $1/M$ for an M-user system). The throughput T increases in the argument D and varies in the interval $[T_{\min}, T_{\max}]$, where $T_{\min}$ is achieved for $D = L$ while $T_{\max}$ is the upper bound reached when the RRA can transmit to the L users having the channel quality equal to the maximum. For any value of $D \in \{L, ..., 2L\}$ (or $\{L, ..., ML\}$ in the general case), the resulting points $(F(D), T(D))$ form a Pareto boundary, and any improvement in a player's outcome necessarily results in a worsening of the other's (note that if s_1 and s_2 coincide then F and T are functions of only one variable, D). For all the details about the game theoretic description of the system we refer to [3], where the complete normal-form representation is given for the non-cooperative case.

Here we take a cooperative approach and describe the interaction between the scheduler and the RRA as a bargaining process. In order to fit our problem within the Nash bargaining framework, we introduce some assumptions:

1) the payoff functions $F(D)$ and $T(D)$ are properly translated and rescaled in the interval [0,1]. We can do that since the NBS is (by axiom) independent of affine transformations. For ease of notation, in the following we still refer to the transformed functions as $F(D)$ and $T(D)$.
2) D is treated as a continuous value.
3) $F, T \in \mathcal{C}^1(\mathbb{R})$, i.e., $F(D)$ and $T(D)$ and their first-order derivatives are continuous functions;
4) when the players propose different values of D, their payoffs vary with continuity in [0,1] forming a convex set upper bounded by the Pareto frontier of all the agreement points (see Fig. 1).
5) the disagreement point is set to $\mathbf{d} = (0, 0)$.

According to the theory we have to find the point

$$(\widetilde{F}, \widetilde{T}) = \underset{(F,T) \in \mathcal{S}}{\operatorname{argmax}} (FT) \tag{2}$$

which means finding the value $\widetilde{D}$ generating the couple $(\widetilde{F}, \widetilde{T}) = (F(\widetilde{D}), T(\widetilde{D}))$; it is worth noting that $\widetilde{D}$ lies on the Pareto frontier and not within the convex set $\mathcal{S}$. From a geometric point of view, the NBS represents the unique point of tangency between the feasible set $\mathcal{S}$ and the generic hyperbola $FT = k, k > 0$. These hyperbolas are the contour lines of the function $z(F, T) = FT$ (see Fig. 1). An advantage of modeling the system as a Nash bargaining problem is that the theory guarantees the existence and uniqueness of the solution, in addition to the fact that this solution represents an *equity* point between the players, as explained in Section II. Sometimes this is also referred to by saying that the NBS realizes the *maximum utility transfer*: moving away from that point, the proportional increment in the payoff of one user is less then the proportional decrement sustained by the other user, thus the overall benefit is negative.

Hereafter we present a possible efficient algorithm to find this solution. In this way we enable the dynamic estimation of the optimal value of D (with respect to the NBS) based on of the current network state, instead of using a value fixed a-priori that should be re-computed every time a variation in the scenario occurs. The algorithm is iterative. The search interval is exponentially reduced, so the complexity is logarithmic. We select some increasing values of D in the initial interval, compute the corresponding points through the Nash bargaining

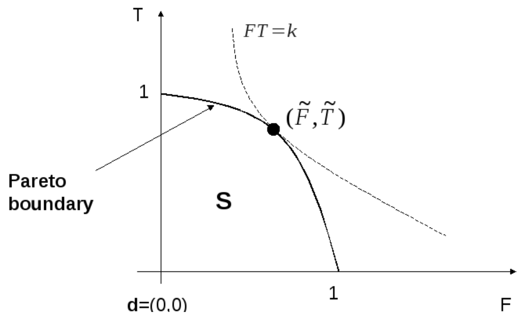

Fig. 1. Pareto boundary and NB solution

function $z(F(D),T(D)) = F(D)T(D)$ and measure the slope of the segments connecting them. Taking into account that the derivative of a $\mathcal{C}^1(\mathbb{R})$ function is positive before the point of maximum and negative after it, we can restrict the interval of interest. For example, if the slopes of the segments are all positive, then D_0 and D_1 must be lower than the point of maximum. Thus, the lower bound of the interest interval can be set to D_1. Similar considerations apply to the other combinations of signs, leading to the decision tree in point 5) below. We iterate until the interest interval is small enough, below a fixed precision ϵ, meaning that we are sufficiently close to the exact value. The steps are the following:

1) Set $a = L$, $b = 2L$. Call $I = b - a$;
2) if($I \leq \epsilon$) then return (int)($I/2 + a$);
3) choose $D_0 < D_1 < D_2$ in the interval $[a, b]$ such that $D_i = a + 1/4 * (i + 1) * I, i = 0, 1, 2$;
4) find the point $P_i = (D_i, z(D_i))$, and determine the slopes λ_1 and λ_2 of the segments $\overline{P_1P_0}$ and $\overline{P_2P_1}$;
5) change the extremes a and b of the interval according to the sign of the slopes. In particular:
   - if($\lambda_1 > 0$) then $a = D_0$;
   - if($\lambda_1 \leq 0$) then $b = D_1$; jump to 6);
   - if($\lambda_2 > 0$) then $a = D_1$; jump to 6);
   - if($\lambda_2 \leq 0$) then $b = D_2$;
6) update I; jump to 2);

Note that the value of D is rounded down to an interger.

Proposition 1: The algorithm above has complexity $\Theta(\log_2(\frac{I_0}{\epsilon}))$, where I_0 is the initial lenght of the interval of interest.

Proof: For any choice of the 3 points D_0, D_1 and D_2, the pairwise distance is $\frac{1}{4}I$. According to point 4), at each iteration at least one of the extremes of the interval is changed and its total length is halved. Therefore, after n steps the length of the interval is $I_n = I_0(\frac{1}{2})^n$. From the inequality $I_n \leq \epsilon$, we obtain the logarithmic complexity stated in *Proposition 1*. ■

VI. Numerical results

We verified the ability of the proposed solution to converge towards a good trade-off between the payoff functions by means of simulation. All the performance indices are characterized by a 95% confidence interval with a maximum relative error of 5%.

To carry out our tests we used the ns3 simulator with an extension for LTE systems described in [18]. We modified the MAC layer by introducing our scheduler and RRA modules.

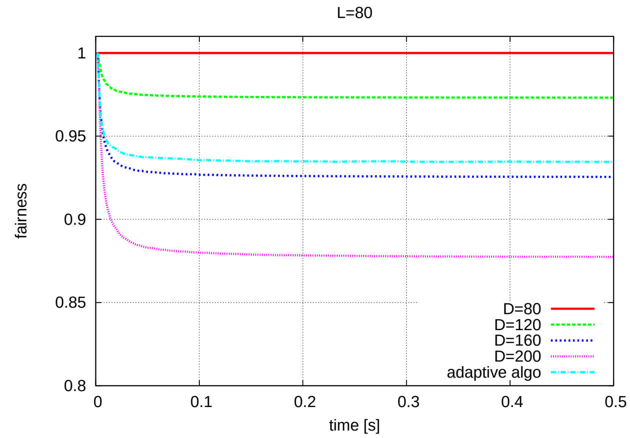

Fig. 2. Fairness for different values of D

The first one adopts a credit-based policy and guarantees fairness by selecting packets from the flow queues according to their residual credit. Flows are assumed to be always backlogged. The second module deals with resource allocation by using a greedy criterion: blocks and packets are matched in order to maximize the total throughput given the channel condition. This information is obtained by the base station through periodic feedback sent from the users of an LTE network. We assume that the channel coherence time is greater than the feedback interval, in our case equal to one subframe duration (1 ms, according to LTE standard). The radio propagation model takes into account the effects of path loss, penetration, shadowing and multipath fading (modeled using Jakes' model [19]). Each resource unit allocable to users has a duration of one subframe and is made of 12 adjacent subcarriers with 15 kHz spacing (equal to one subchannel of 180 kHz). We considered a total of 80 frequency subchannels, plus 20 subchannels for the uplink, for a total of 20 MHz bandwith according to what indicated in the standard [4]. The scheduling and allocation decisions are made at the beginning of each subframe. The main simulation parameters are reported in Table I.

In Fig. 2 and Fig. 3 the two normalized payoff functions are represented versus time for several values of D. The trade-off expected from the theory is confirmed: once the value of L is fixed, the fairness decreases in D while the throughput increases. From a quantitative point of view, the variation depends on several factors, e.g., the number of users in the cell, the channel conditions, the transmission power, the number of available subchannels. For the sake of completeness, we ran additional simulations by changing some of these parameters.

Parameter	value
number of flows	2
packet size	500 bytes
number of subchannels for the downlink	80
number of subchannels for the uplink	20
frame duration	10 ms
subframe duration	1 ms
transmission power	43 dBm

TABLE I
MAIN SYSTEM PARAMETERS

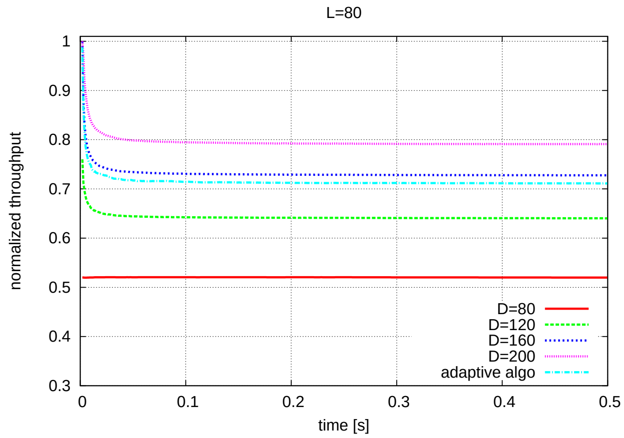

Fig. 3. Throughput for different values of D

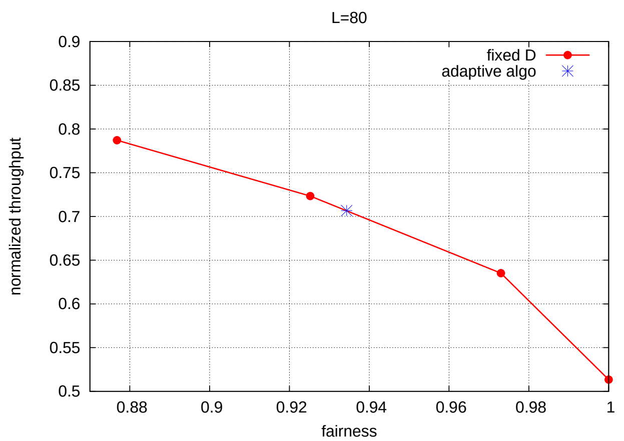

Fig. 4. Pareto boundary of the game and operating point of the algorithm

We cannot report all the results in this paper because of space constraints, but in every case the mutual relations among the curves for different D were the same.

Both in Fig. 2 and Fig. 3 we can note that the proposed algorithm for the estimation of D leads to an intermediate value of both performance indices. The two functions cannot be jointly maximized. Indeed, what is maximized is a common utility function, represented by the product of each player's payoff. This situation is summarized in Fig. 4, where both the Pareto boundary and the NBS point are drawn. This point lies on the frontier, as expected from the theoretical analysis.

We think it is worth stressing again that the main point of the proposed solution is its adaptivity. One could think of fixing D a-priori, after a preliminary study, but the system performance is context dependent, thus the value should be re-computed every time a change occours, which is not practical.

VII. Conclusions

In this paper we have addressed the problem of resource allocation in the downlink of an LTE cellular network. A possible design approach has been introduced for a modular and flexible system with cross-layer information. A formal model has been proposed which makes use of the Nash Bargaining theory, where a cooperative approach guarantees the existence of a fair and efficient operating point. A feasible and efficient algorithm to reach this point by dynamically setting a parameter has been introduced as well. We ran some simulations to validate our analysis by using a realistic LTE model built with the well known ns3 network simulator. The results confirm the optimality of our solution and its adaptability to changes in the scenario.

Further developments of this work include the extension to the multicell case, where our intra-cell scheme should be integrated with resource allocation strategies among the base stations. Even more interesting is the case of inter-operator spectrum sharing, where each base station is supposed to keep some private information, thus leading to an incomplete information system.

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