

# Crowdsensing Strategies Inspired by Choir Management Analyzed via Game Theory

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**Abstract**—In this paper, we draw an analogy between crowdsensing scenarios and the real life activity of singing in a choir. We identify some similarities, in particular for what concerns the role of the network coordinator and the choir director, as well as the common desirability of eliminating non-collaborative behavior (free-riding). Inspired by this comparison, we identify some strategies that the “director” can implement during “choir rehearsals” and we give a game theoretic analysis of their effectiveness. The general model is based on characterizing the willingness to undertake effort for the common task as a user’s private type, which is compared to the contribution cost to decide whether to contribute or free-ride. Imposing some access penalty is known to reduce significantly the onset of free-riding, and we discuss possible ways to implement such a penalty, namely, we compare a probabilistic exclusion of free riders, as well as a multiplicative and an additive penalty to access, and we show the better effectiveness of the last strategy.

**Index Terms**—Peer-to-peer computing, Crowdsourcing, Game theory, Mechanism design, Choir singing.

## I. INTRODUCTION

THE CROWDSENSING paradigm encompasses multiple models by which individual communication devices share data and collectively extract useful information towards a common goal [1]. This finds special application in the Internet of Things, whose services can be greatly extended to exploit ambient intelligence. Possible applications include wide-scale monitoring of chemical and acoustic environmental pollution, supervision of city infrastructure, or description of social phenomena such as road traffic or crowd gathering [2]. More recently, crowdsensing approaches have been proposed to observe the state of global scale pandemics (such as covid-19) and alert individuals who become at risk of getting the disease due to proximity with an infected subject [3].

We see a similarity between this technological scenario and the real-life experience of singing in a choir. Music production, and in particular choir singing, has been shown to produce positive influences on an individual’s well-being from the physical and psychological standpoints, with an overall decrease in stress, depression, and anxiety [4]. At the same time, group participation implied by the choir activities is a positive experience for social aggregation [5]. Thus, singing in a choir can be compared to crowdsensing for its main result (musical production) is achieved through the combined efforts of individuals who, if acting alone, would not obtain the same

TABLE I  
ELEMENTS OF A CROWDSENSING SCENARIOS AND THEIR COUNTERPARTS  
IN A CHOIR SINGING

Crowdsensing	Choir
Free riders	Underperforming participants
Network crowdsourcer	Choir director
Sensing tasks	Rehearsals
Incentives/punishments	Techniques for increasing effort

result. Also, singers enjoy the experience in the same way as individual crowdsensers can benefit from the service.

Another similar aspect is that the performance of crowdsensing systems can be negatively affected by participants that do not significantly contribute to the collective task and therefore waste communication resources and slow down the execution of the task [1]. This is a common problem to many peer-to-peer systems, who have designed mechanisms to counteract the behavior of users not contributing to the entire collective task but possibly enjoying the benefit of its execution by other participants, generally called *free-riders* [6]. Depending on the peer-to-peer system at hand, the problem of free-riding can arise from several issues, including the selfish nature of some users, but also the different ability by the individual participants to contribute to the task [7].

Musical ensembles, such as orchestras, choirs, and bands, can be seen as a society in miniature [8], in which individuals contribute voluntarily, through effort in rehearsals, for the achievement of a collective outcome, which is the performance during concerts. However, being a social peer-to-peer system, choirs are not immune to free-riding, i.e., opportunistic behaviors where a person benefit from the global result in spite of putting little effort, or no effort at all, at the expense of collective welfare [9].

In choral practice, there are well known situations where wrong actions from one chorister may cause more distress for the rest of the section or the choir. For example, a chorister may not be putting enough effort (due to physical limitations or lack of commitment) and therefore overload her section of the choir or even undermine the stability of the entire ensemble. On the other hand, she could be overenthusiastic in singing, thus ruining the balance of the choir. Or finally, a chorister that is neither below nor above its expected contribution to the group can eventually be absent at the rehearsals before the concert for personal unavailability.

Some analogies between crowdsensing tasks and the activity of a choir are detailed in Table I. Inspired by them, we explore the crowdsensing scenario with such a mindset to present

a game theoretic characterization of free-riders and discuss practical policies that the choir director usually implements to increase the effort of the choristers. In particular, we consider a basic game taking place during “rehearsals” (i.e., sensing attempts) and we capture the mechanisms and parameters at play. Further, we discuss mechanisms to implement in the crowdsensing system to encourage voluntary participation.

One common option, also paralleled in many peer-to-peer communication systems, would be the exclusion from concerts for free-riders: while being very efficient, this solution strongly relies on the surveillance capability of the crowdsourcer, namely the probability of detecting free riding behavior. In many scenarios, this option is not available; this can happen either in a real choir, whenever voluntary participation is encouraged for social reasons, even when the actual contribution to the performance is limited, or in real crowdsensing scenarios, where the inability to contribute to the task may likely originate to bad channel conditions and/or battery outages, rather than an intentional free riding.

As a second option, we discuss affecting the individual benefit of participation, either by decreasing the individual rewards or adding some participation costs. The motivation behind this action is that free riding behavior is made inconvenient or not sustainable. As a result, elements that are unable to contribute leave voluntarily, instead of being forced to do so, without any need for being individually monitored for free-riding. If a share of free-riders prefer to leave the choir than receiving a diminished benefit, the remaining choristers would see a relieved load and the director would interact with a more reliable ensemble of performers.

Hence, in this paper we compare three different strategies, which can be figuratively represented in the practical context of a choir as (i) excluding from the performance choristers who underperformed in rehearsals; (ii) reducing the appraisal of the director to the choristers; and (iii) increasing the length or intensity of the rehearsals. These penalties to free-riding are differently framed in the mathematical formulation: (i) is considered as a probabilistic penalty; (ii) is a certain multiplicative penalty to the utility, while (iii) is an additive penalty. Through a mathematical discussion, we will argue that (iii) is actually more effective in controlling free-riding.

The rest of this paper is organized as follows. In Section II we consider the general framework for a crowdsensing scenario. Section III discusses the implementation of penalties for free-riding, inspired by choir dynamics. In Section IV we present some numerical results. Finally, Section V concludes the paper and gives some insight about possible future work.

## II. SYSTEM MODEL

Crowdsensing relates to data acquisition with the involvement of many individuals to contribute their sensed data [1]. This is the consequence of the pervasive integration of sensing capabilities in many computing devices at the edge of the Internet, which reflects the evolution towards the so-called Internet of Things [2].

Such a scenario relies on voluntary participation of the involved “data sensors,” which makes it generally fall under

the umbrella of distributed architectures and most specifically peer-to-peer systems. Despite some different nuances that can be encountered, such as the distinction between participatory or opportunistic sensing [10], we will describe the entire crowdsensing procedure from a general perspective. In particular, as argued in the introduction, we compare the crowdsourcer to the director of a choir, that is trying to extrapolate data (i.e., musical harmony) from multiple individuals, who voluntarily commit their effort, at least in principle, and enjoy some common reward for their performance.

Clearly, this scenario is vulnerable to free-riding, which in our case declines as submission of low-quality data to individuals that do not want to spend too much effort in the task, but still enjoy the shared reward [7]. Similar to what happens in the choir, this kind of actions puts distress on the remaining sections of the participants that have to increase their effort.

We analyze the crowdsensing scenario under a general framework for peer-to-peer systems inspired by [9], in which users’ *generosity* is taken into account as the user’s Bayesian type [11]. The heart of the model consists of a characterization of peer-to-peer users as rational agents, with a private and intrinsic characteristic parameter  $\theta_i$  for each user  $i$ , reflecting the willingness of that user to commit resources to the collective task. In social contexts, this can be intuitively thought of as a quantitative measure of decency or prodigality. For the sake of mathematical tractability, we consider  $\theta_i$  to be between 0 and 1, respectively taken as the most selfish type (never contributes) or the most selfless one (always contributes no matter what). In general, users decide whether to contribute or free-ride based on the relationship between the cost of contribution and their types.

We assume that the cost of contributing is proportional to the *reciprocal* of the total share of contributors, because when many people free-ride, the load on contributors increases [9]. This is also a typical feature of choir singing. Moreover, we denote as  $a > 1$  the theoretical benefit of contributing to a successful performance when everybody contributes. Collecting these assumptions, we take the total net utility for contributors to be equal to benefit minus costs, i.e.

$$W_C = ax - \frac{1}{x} \quad (1)$$

where  $x$  represents the fraction of the contributing users. Notably, the cost of contributing has been taken with coefficient equal to 1, but this is not restrictive, in light of the utility function having just descriptive ordinal value (the higher, the better). Still, there is a parameter in the definition, i.e., the value  $a$ , that can be changed to tune the desired utility value. We also remark that for the system to be feasible, there must be at least somebody contributing; if  $x$  tends to 0, the utility of the (few) contributors tends to minus infinity.

To compute  $x$ , we can exploit the Bayesian framework of user types. First of all, it is immediate to prove that users follow a threshold policy, i.e., they contribute only if their type is large enough, and beyond a given threshold. This is because if a user with type  $\theta_0$  contributes, also any other user with type  $\theta > \theta_0$  is also willing to contribute [11]. So, there

must exist a threshold  $t$  for which the decision of a rational user with type  $\theta_i$  is:

$$\begin{aligned} \text{Contribute} & \quad \text{if } \theta_i > t \\ \text{Free-ride} & \quad \text{otherwise} \end{aligned} \quad (2)$$

This minimalistic framework is already enabling some interesting implications. The value of  $x$  is determined [9] by considering the type distribution, that is,  $x = \Pr(\theta_i \geq t)$ .

A useful assumption that just makes the following computation simpler is that of a uniform distribution of user types, so that  $\theta_i \in \mathcal{U}([0, 1])$ , resulting in  $t = 1 - x$ . In other words, the cost threshold is also equal to the share of the free-riders in the entire network population. We will therefore make this assumption just for the sake of simplifying the computations, but we emphasize that this hypothesis can easily be relaxed just at the price of more cumbersome equations.

For what concerns free-riders, we assume that they control the value of  $x$  through their collective action, since indeed, the more individuals adopt a free-riding behavior, the lower  $x$ . However, the value  $W_C$  from (1), which was used to describe the global utility of contributing users, is not appropriate to capture that of free-riders, since it would turn out that the maximal utility would be achieved for  $x = 1$ , which is true for contributing users, but not for the free-riders. Instead, free-riders neither have to bear the cost proportional to  $1/x$ , nor desire to increase the share  $x$  with their action.

In the following, we will assume this total utility  $W_F$  for free-riders, quantified as

$$W_F = a(1 - x) \cdot \mathbb{1}[W_C] = a(1 - x) \cdot \mathbb{1}\left[ax - \frac{1}{x}\right] \quad (3)$$

where  $\mathbb{1}[\cdot]$  denote a unit step, that is  $\mathbb{1}[y] = 1$  for  $y \geq 0$  and 0 otherwise. This definition captures that the free-riders get a higher utility when  $x$  decreases, which is implied by their benefit being their share of the overall benefit, that is, the value  $a$  times the free-rider share of  $1-x$ . Also, the free-riders do not pay further costs, since they just enjoy the benefit of participating in the system without putting effort. However, in order for the existence itself of the free-riders to be sustainable, the utility of contributors must be greater than 0. This is why we also include a unit-step term to signify that in the case the contributors achieve a negative utility, they leave the system and there is no overall benefit  $a$  to share.

We can give an extensive form representation, as depicted in Fig. 1, of the selection dynamics involving the users. Assume that this decision tree is followed by each player, and we overall have  $N$  players. Also assume that contributing to the system has a cost  $c$ . It can be seen that to keep the notation coherent with the previous assumptions, we can set  $c = 1/x^2$ . On the top node, the virtual player *Nature* chooses the type of the player as a *Contributor* or a *Free-rider*, so that the shares of these two types are  $x$  and  $1-x$ , respectively. After this choice, the Chorister can decide whether to Contribute or Free-ride. Notably, this choice is not actually determined a priori from the type, but rationally implied by the utilities. Specifically, we can represent a willing contributor as a chorister who puts utility 0 in Free-riding. Conversely, a Free-rider plays move F just because egoistically compares the choice of two different

utilities. Provided that  $W_C$  is non negative, the preferred choice happens to be to play F, since it does not imply any cost  $c$ .

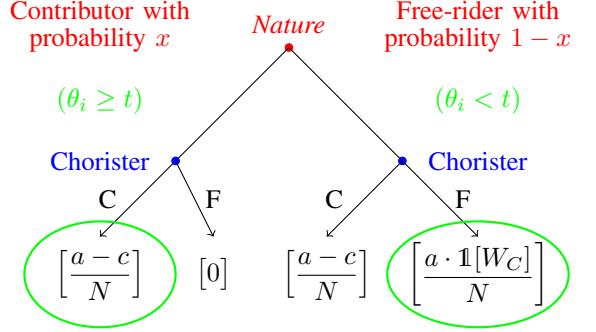


Fig. 1. The rational choice for a single chorister, according to her type.

Even though in the extensive form of Fig. 1 the probability  $x$  should be drawn at the beginning, we see that the value of  $x$  ultimately affects the payoffs in the end node. Thus, we can set a further game-theoretic perspective, where it is actually the free-riders, as a matter of fact, that choose the value  $x$ , and do so selfishly, which implies that  $x$  is minimized under the constraint that  $W_C$  is greater than or equal to 0. Since  $W_C$  is increasing in  $x$ , such a condition immediately leads to driving  $W_C$  to 0 and setting  $x = 1/\sqrt{a}$  as the best choice for the free-riders. This can also be seen as a way to set the value of  $a$ , since we need  $a = 1/x^2$  to have a contributing share equal to  $x$ . For example, if  $a = 1.5625$  we obtain that contributing individuals are  $x = 1/\sqrt{a} = 80\%$ , while if  $a = 100$ , they only amount to 10%.

We also remark that the game theoretic property that under selfish objectives of the nodes, the overall utility of contributors is zero, was kinda expected and well known in the literature [12]. However, such a situation is particularly unpleasant from the standpoint of the crowdsourcer, that can be seen as the choir director, since all the utility is in the end assigned to free-riders, while honest contributors get nothing in return. To solve this problem, we identify some possible solution mechanisms in the next section.

### III. PENALTY MECHANISM DESIGN

Taking inspiration from the aforementioned analogy with the dynamics of a choir, we identify and discuss three possible practical strategies for diminishing the impact of free-riders in the system. In particular, the following countermeasures are typically adopted in real life.

- 1) If free-riders can be detected, i.e., information about the individual types is available, the director can decide to exclude them from the choir (the sensing group).
- 2) The director can act angrily to scold the choristers, in the effort of discouraging free-riders.
- 3) The director can impose an entry penalty (e.g., long and tiresome rehearsals) to the choir participants.

In the following subsections, will review these strategies, by discussing how to mathematically formalize them, and assess their effectiveness in contrasting free-riders.

### A. Exclusion from the system

If we had perfect information about the type of each individual user, we could exclude the users of the lowest Bayesian type to increase the contribution level. However, this exclusion mechanism might be questionable and suffer from major problems. First of all, user types may not be observable in practice. Moreover, even when user types are observable, excluding potential contributors simply based on their innate type rather than their actual behavior precludes the possibility of rational decision-making [9]. Thus, it is probably more sensible to introduce some *penalty* mechanism to reward contributing behavior.

Still, an exclusion mechanism can be designed under the assumption that free-riding behavior is observable, even though innate user types may not be; that is, it is possible for the crowdsourcer to label users as either contributors or free-riders, and assign a penalty to users of the latter type. The simplest way to implement such a penalty would be exclusion with probability  $p$ . When free-riders are excluded, they get nothing, so their expected utility becomes  $(1-p)W_F$ . We remark that this probability can also be regarded as a deterministic penalty resulting from service differentiation, under which the benefit of free-riders is somewhat reduced by a fraction  $(1-p)$ , while contributor benefits are not. This approach would be totally equivalent from the game theoretic standpoint to the previously mentioned probabilistic version since game theory operates according to *expected utilities*. Downgrading the performance of the free-riders has two effects, both of which lead to a higher contribution level. First, since free-riders get only a fraction  $(1-p)$  of the benefits, the load placed on the system decreases to  $x + (1-x)(1-p)$ ; therefore, contribution cost becomes  $(x + (1-x)(1-p))/x$ , which is lower than the previous cost  $(1/x)$  in which the contributors had to work for the whole group. Second, the penalty introduces a threat, since users who free-ride know that they will get reduced service. Under this penalty mechanism, the utilities  $W_C$  and  $W_F$  change from (1) and (3) and become

$$\begin{aligned} W_C &= a \frac{x}{x+(1-x)(1-p)} - \frac{x+(1-x)(1-p)}{x} \\ &= a \frac{x}{1-p+px} - \frac{1-p+px}{x} \end{aligned} \quad (4)$$

$$\begin{aligned} W_F &= pa(1-x) \cdot \mathbb{1}[W_C] \\ &= pa(1-x) \cdot \mathbb{1}\left[a \frac{x}{1-p+px} - \frac{1-p+px}{x}\right] \end{aligned} \quad (5)$$

and the contribution share  $x$  is derived as

$$x = \frac{1-p}{\sqrt{a} - p} \quad (6)$$

It is therefore implied that excluding nodes from the network is not always an adequate strategy to reach the goal of limiting free-riding, for two main reasons. The first one lies in its practical requirements. Differently from the other strategies that we will discuss in the following, this penalty requires monitoring of the nodes behavior to detect free-riders. Such a feature is not only costly, and prone to errors, but also potentially harmful for the network when it leads to remove

nodes that are just unwillingly and temporarily unavailable to contribute. This would be the case of excluding a chorister from the final performance just because she skipped some rehearsals due to a cold, or similarly to remove a node from the sensing set just because it was experiencing a non-permanent outage, e.g., due to channel fading or battery exhaustion.

The second reason is of game theoretical nature. The analytical result of (6) implies that excluding nodes that are detected to be free-riders instead of reducing the share of free-riders in the network actually increases it. This is because in our game theoretic setup free-riders are assumed to be anticipators rather than takers [13]. In other words, nodes act strategically and, being aware that free-riding is punished with exclusion and therefore the share of contributors is theoretically higher, they are disincentivized to put effort to the common goal. Especially, we observe that when  $p$  tends to 1, that is, the free-riders are always discovered, our game theoretic setup actually implies that  $x$  tends to 0. The specific reason for this awkward result is that the free-riders increase their activity very much when  $p$  is high, in the effort of getting a non-zero payoff. Since in the end the free-riders just aim at nulling the payoff of the contributors [12], they can do so more easily if the share of contributor is increased by eliminations. All these apparently counter-intuitive results are in reality consequences implied by the rationality assumption in the behavior of the nodes. Indeed, they are also confirmed by other game theoretical frameworks, especially in relationship to security and avoidance of malicious behavior. For example, in [15] it is found that an increased surveillance causes malicious nodes to disturb the network operations more frequently. This is once again a consequence of the strategic planning by malicious nodes only aimed at maximizing the payoff.

### B. Multiplicative penalty: scolding the choir

A simpler intervention for the director would be to scold the choir in the attempt to discourage free-riders. We can model this strategy as a replacement of the total choir utility  $a$  with another value  $b$ ,  $1 < b < a$ , while the contribution cost is left unchanged. We consider it to be a *multiplicative* penalty, since we can define  $b$  by taking it as a fraction of the original value  $a$ . By repeating the analysis of Section II, we can easily see that the selection mechanism of contributing versus free-riding stays the same as it only depends on the user's type. The only modification is the lower value of  $b$  that leads to a higher contributing rate of  $x = 1/\sqrt{b}$ ; in other words,  $x$  is multiplicatively increased.

However, this apparent improvement is questionable from a game theoretic perspective. We can see this in more detail by formalizing a 2-player game where the (D)irector plays against potential (F)ree-riders, treated as a single player who determines the value of  $x$  as its move. In this setup, player F is just driven by utility  $W_F$ , while we assume that player D is concerned about the payoff of contributing choristers and therefore her payoff is  $W_C$ . Moreover, player D has two actions available: "get Angry" and "do Nothing." The latter gives the exact model described in Section II with parameter  $a$ . Instead, getting Angry replaces the value of  $a$  with  $b$ .

We can see that, in this game, player F can still easily drive the payoff of player D to 0 for both cases. Moreover, the effectiveness of playing "get Angry" is questionable as it would be a dominated strategy for player D meaning that, regardless of the choice of the other player, it never yields a bigger payoff with respect to another strategy, which in this specific case it is "do Nothing." In game theoretic setups, when this dominance is strict, the dominated strategies can be safely ignored [14] since they are never chosen by rational players. in our case, the dominance is not strict, yet, the usefulness of this strategy is very limited. Furthermore, even if a bigger contribution level were reached by player D choosing to "get Angry," it would never exceed  $x = 1/\sqrt{b}$ , hence the share of contributors would never be 100% since  $b > 1$ , which implies that the payoff for the contributors to the choir can never be bigger than 0. On the other hand, free-riders are still able to get a positive utility, albeit lower. Indeed, there is no sensible way for the director to push the utility of the free-riders below 0; the only option would be to make the entire maximum utility, also including the contributors, to be negative, which would make the game meaningless.

Thus, despite its simplicity of implementation, and even its ability to achieve a bigger value for  $x$ , such a multiplicative penalty is insufficient to fully pursue the desired objectives. This would be true in both a real-life system of a choir and in a crowdsensing scenario, which is why we also look for more penalty options.

### C. Additive penalty: longer rehearsals

We now introduce another action that the Director can undertake, that is to perform longer and tiresome rehearsals. This is mathematically captured by assuming a fixed and non-excludable *additive* penalty  $d$  to be actually subtracted by the utility of all the choir participants (not only free-riders, but also contributors). This leads to the following modifications in (1) and (3):

$$W_C = ax - \frac{1}{x} - d \quad (7)$$

$$W_F = \left( a(1-x) - d \right) \cdot \mathbf{1} \left[ ax - \frac{1}{x} - d \right] \quad (8)$$

Once again, if  $x$  is ultimately determined by the action of potential free-riders that decide not to contribute, its value would turn out to be a solution of  $ax^2 - dx - 1 = 0$ . Such a second-degree equation has only one positive equation that, after some manipulations, can be re-worked as

$$x = \frac{1}{\sqrt{a}} + \frac{d}{2a} + \frac{d^2}{8a^{3/2}} + \frac{o(d^2/\sqrt{a})}{2a} \quad (9)$$

Notably,  $x$  is increased from the initial value of  $1/\sqrt{a}$  and the increase is not only the linear term  $d/(2a)$  but there are other non-linear terms that make the increase steeper. We can conclude that such a strategy is potentially more effective to discourage free-riding in both scenarios (choir rehearsals and crowdsensing), which is also in agreement with other findings in the general field of peer-to-peer systems [6], [9].

In the next section, we will better check this effectiveness with numerical examples and also discuss another important

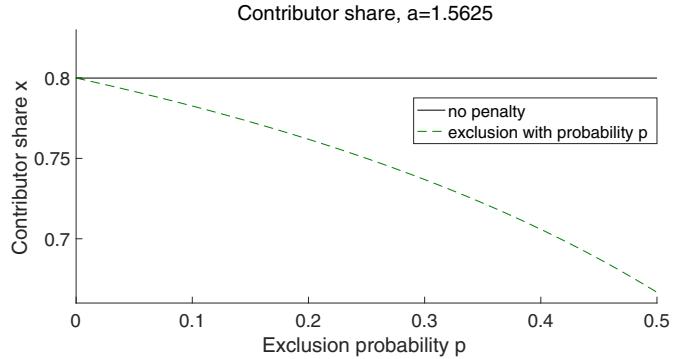


Fig. 2. Contributor share  $x$  under exclusion probability  $p$  of free-riders.

feature of the additive penalty, namely, it is also capable of pushing the utility of free-riders below 0. This is actually an important aspect, since we may argue that the harmful effect of free-riders is that of nulling the welfare of the contributors [12] and this is sustainable only as long as their own payoff is greater than 0. Since in our framework utilities only have an ordinal meaning, if the maximum achievable utility for the free-riders is negative, that would still be the best choice available to them. But in a practical setup, this would imply that the free-riders prefer to leave the system rather than staying. It would make more sense to assume a further option for the free-riders (i.e., to leave the system) and impose that leaving the system gives zero payoff.

Therefore, we can consider the outcome where free-riders get a negative payoff as representing a situation where they are incentivized to voluntarily leave the system, as opposed to being thrown out forcedly as in the policy of Section III-A, which would not require any ability for the crowdsourcer (i.e., the choir director) to track the individual effort of the participants. It is simply a natural consequence of the Bayesian type of the free-riders to be less inclined to generosity, and therefore more negatively affected by a decrease in the utility.

## IV. NUMERICAL RESULTS

To numerically test the proposed techniques, we compute the derived equations with practical values. In particular, in all cases we consider  $a = 1.5625$  that gives an initial contributor share of  $x = 0.8$ .

First, we evaluate the proposed strategy of Section III-A, i.e., exclusion of free-riders with probability  $p$ . Fig. 2 shows the share of contributors  $x$  computed according to (6). We see that this mechanism leads to a lower value of  $x$  as  $p$  increases, which is in accordance with the motivations given earlier. Thus, this result simply confirms that such a policy may be inadequate to control the free-riders and can backfire by actually increasing their presence.

Next, we consider a multiplicative penalty according to the discussion of Section III-B and an additive penalty following Section III-C. These policies were found to be more effective in reducing the value of  $x$ . In particular, the multiplicative penalty can very easily set the desired share of free-riders by choosing the value of  $b$ . For our numerical examples, we considered  $b = (1 + a)/2 = 1.28125$ . Instead, inserting an

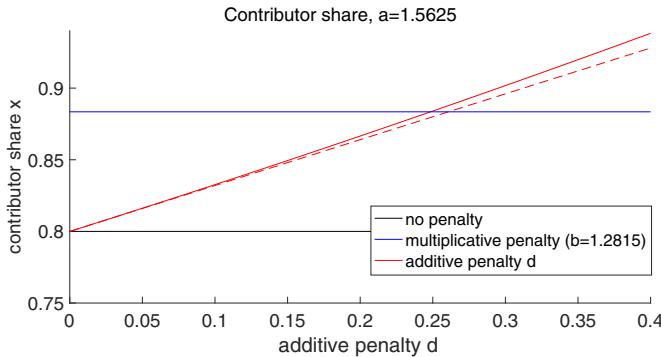


Fig. 3. Contributor share  $x$  under multiplicative and additive penalties. The red dashed line represents (9) with just the linear terms.

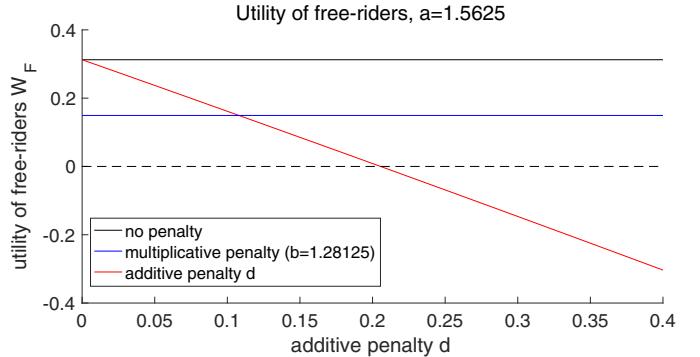


Fig. 4. Utility of free-riders  $W_F$  under multiplicative and additive penalties. The black dashed line is utility 0, plotted for comparison.

additive penalty requires more finetuning and for this reason we considered the value of  $d$  as the independent variable in the following plots.

Fig. 3 shows the share of contributors  $x$  with an additive penalty  $d$ , versus the value of  $d$  itself, as per (9). The performance of the multiplicative penalty is also shown for comparison. We can see that an increased penalty causes free-riders to exit the system; actually, in the model they just determine a higher  $x$ . The increase is approximately linear; see the dashed line in the figure, plotting the linear trend of  $x$  by omitting all the non-linear contributions in (9).

Fig. 4 computes instead the payoff of the free-riders according to (8). This is also compared with the original payoff of the free-riders from (3) and the same value but considering  $a$  to be replaced by  $b$ , as per the multiplicative penalty strategy. We remark that the payoff  $W_C$  of the contributors is not shown, since it would be 0 under any policy, as per the discussion of the previous sections. This is an intrinsic aspect of game theoretic models where users are anticipators only driven by selfish goals [12], [13].

At any rate, the numerical results show the advantages of the additive penalty strategy of Section III-C. First, it can practically increase the value of  $x$  and even go beyond any multiplicative strategy. Second, it can make the payoff  $W_F$  of the free-riders to go below 0.

Still, there are some limitations in the model that need to be addressed. In particular, adding a penalty to the users for being part of the choir (the crowdsensing system) also penalizes the contributors. Even though this is not reflected by a decrease in  $W_C$ , since, as said, it is 0 anyways, a practical model should also consider an increased dissatisfaction in the contributors because of the penalty, especially since it is introduced to deter the free-riders but it ends up in penalizing everyone. In the model considered in the present paper, contributors still accept the system benefit anyways, since their type dictates so; see Fig. 1, where their only practical choice is to contribute to the system. Still, a wider model where also contributor can intervene and make rational decision, possibly reflecting their dissatisfaction with the additive penalty, would certainly be possible and worth investigating in future works.

## V. CONCLUSIONS AND FUTURE WORK

We constructed a game-theoretic model of user behavior in crowdsensing scenarios, based on the similarities with the practice of choir singing. We discussed how participants can be adopt either a contributor or free-rider behavior, based on their Bayesian type, and used this model to discuss possible ways to penalize free-riders so as to improve the overall system performance. In practice, these penalties threaten potential free-riders and induce them to contribute, or leave the system.

We reviewed three mechanisms aimed at disincentivizing free-riders, namely, a probability of exclusion for free-riders, a multiplicative penalty to the entire system utility, and an additive penalty imposed to all participants. Of these, the last one seems to be the more effective, at least within a game-theoretic setup.

We remark that, for the sake of simplicity, this paper just discusses all the game theoretic scenarios as static games. A possible further development involves a dynamic game representation, where the option to join or leave the system (i.e., the choir) is also made explicit, and discuss whether the equilibrium points found by the present analysis reflect in the dynamic outcome of the system through repeated interactions, which would give a practical way to implement the techniques discussed in this paper with iterative mechanisms. Moreover, such a dynamic representation would add another layer of realism also in the context of discussing the choices of the contributors, instead of just limiting rational decisions to the free-riders. All of these issues are certainly worth investigating in future extensions.

## REFERENCES

- [1] R. K. Ganti, F. Ye, and H. Lei, "Mobile crowdsensing: current state and future challenges," *IEEE Commun. Mag.*, vol. 49, no. 11, pp. 32-39, 2011.
- [2] J. Liu, H. Shen, H. S., Narman, W. Chung, and Z. Lin, "A survey of mobile crowdsensing techniques: A critical component for the internet of things," *ACM Trans. Cyber-Phys. Sys.*, vol. 2, no. 3, pp. 1-26, Jun. 2018.
- [3] J. Cano, J. Cecilia, E. Hernandez-Orallo, C. Calafate, and P. Manzoni, "Mobile crowdsensing approaches to address the COVID-19 pandemic in Spain," *IET Smart Cities*, vol. 2, no. 5, 2020.
- [4] G. Kreutz, S. Bongard, S. Rohrmann, V. Hodapp, and D. Grebe, "Effects of choir singing or listening on secretory immunoglobulin A, cortisol, and emotional state," *J. Behav. Medic.*, vol. 27, no. 6, pp. 623-35, Dec. 2004.

- [5] G. A. Dingle, C. Brander, J. Ballantyne, and F. A. Baker, "To be heard: The social and mental health benefits of choir singing for disadvantaged adults," *Psych. Music*, vol. 41, no. 4, pp. 405-21, Jul. 2013.
- [6] L. Ramaswamy and L. Liu, "Free riding: A new challenge to peer-to-peer file sharing systems," *Proc. IEEE ICSS*, pp. 10, 2003.
- [7] S. K. Madria, A. Mondal "Crowdsourcing: Dynamic data management in mobile p2p networks," *Proc. IEEE ICMDM*, pp. 364-367, 2012.
- [8] J. A. Govias, "The five fundamentals of El Sistema," *Can. Music Educ.*, vol. 53, no. 1, pp. 21-23, 2011.
- [9] M. Feldman, C. Papadimitriou, J. Chuang, I. Stoica, "Free-riding and whitewashing in peer-to-peer systems," *IEEE J. Sel. Ar. Commun.*, vol. 24, no. 5, pp. 1010-9, May 2006.
- [10] N. D. Lane, E. Miluzzo, H. Lu, D. Peebles, T. Choudhury, and A. T. Campbell, "A survey of mobile phone sensing," *IEEE Commun. Mag.*, vol. 48, no. 9, pp. 140-50, Sep. 2010.
- [11] A. V. Guglielmi and L. Badia, "Bayesian game analysis of a queueing system with multiple candidate servers," *Proc. IEEE CAMAD*, pp. 85-90, 2015.
- [12] J. Lu, Y. Xin, Z. Zhang, X. Liu, and K. Li, "Game-theoretic design of optimal two-sided rating protocols for service exchange dilemma in crowdsourcing," *IEEE Trans. Inf. Forens. Sec.*, vol. 13, no. 11, pp. 2801-15, 2018.
- [13] P. Chakraborty and P. P. Khargonekar, "Flexible loads and renewable integration: Distributed control and price of anarchy," *Proc. IEEE CDC*, pp. 2306-12, Dec. 2013.
- [14] A. V. Guglielmi and L. Badia, "Analysis of strategic security through game theory for mobile social networks," *Proc. IEEE CAMAD*, pp. 1-6, 2017.
- [15] L. Badia and F. Gringoli, "A game of one/two strategic friendly jammers versus a malicious strategic node," *IEEE Netw. Lett.*, vol. 1, no. 1, pp. 6-9, Jan. 2019.