A Stochastic Model for Age-of-Information Efficiency in ARQ Systems with Energy Harvesting

Laura Crosara [a] and Leonardo Badia[a]
[a]Dept. of Information Engineering, University of Padova, Italy, email: laura.crosara.1@studenti.unipd.it, leonardo.badia@unipd.it

Abstract

We elaborate a stochastic model of an automatic repeat request transmission system exploiting energy harvesting. We consider an information source powered by a rechargeable battery exploiting a renewable energy source that is using retransmission-based error control. For this system, the transmission policy that minimizes the age-of-information associated with correctly delivered packets is investigated, also analyzing when it is more convenient to retransmit a data packet whose previous transmission attempt was unsuccessful or to send a new one instead. The model is implemented and solved as a discrete-time finite state Markov chain. The behavior of this process is studied under different parameters to characterize some general rules for this kind of system.

1 Introduction

Personal mobile communications are nowadays ubiquitous, which has led to a pervasive diffusion of wireless devices connected to the Internet. This trend has implications in the development of context-aware communication techniques, as well as technological advancements for portable batteries and the exploitation of renewable energy sources to obtain energy harvesting devices (EHD). However, the advancement of chemical engineering in developing new battery technology still lags behind due to cost and material constraints, making this component both a propeller and an inhibitor of large-scale deployment of wireless mobile devices [1].

This results in a push towards the investigation of energy-efficient management transmission techniques, which enables transmission among portable wireless devices even when energy resources are scarce. A further factor that is becoming increasingly important is the concept of age-of-information (AoI), introduced in [2] to quantify the freshness of the information on the status of a remote system. Very often, studies in this area are applied to the development of wireless devices [3], especially for remote sensing. This is especially needed since this kind of devices are frequently placed in hard-to-reach places and therefore often impossible to power. The availability of energy, which varies over time, and the constraints on the batteries can make the exchange of information between the transmitter and the receiver complex, especially if the information requested must be frequently updated. Therefore, it is interesting to analyze a system that is aware of the energy availablity while seeking to minimize the average AoI [4].

Recent publications [5, 6, 7] discuss the AoI in energy harvesting systems, which use natural renewable resources as an energy source allowing the device to ideally operate for an indefinite time. Therefore, in energy harvesting systems, another parameter must be taken into account: the rate at which energy arrives from the external environment and recharges the battery. A system where energy harvesting is well managed cannot consume more power than the harvesting source can provide, in the long run [8].

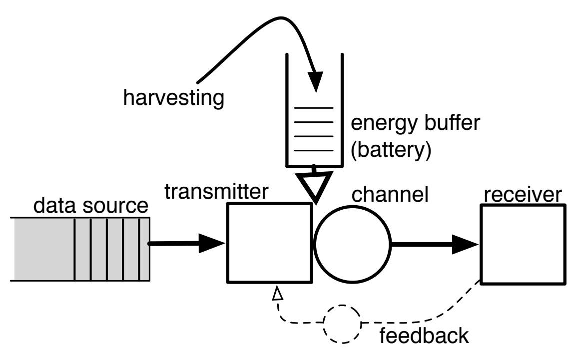

Figure 1 System model

We investigate optimal policies that operate for this purpose, integrating the system with an automatic retransmission mechanism for error control [9]. Our model is summarized in Fig. 1, in which a transmitter, equipped with an EHD which recharges a battery with limited capacity, sends packets to a receiver. This, in turn, may inform the transmitter via a feedback channel of the correct reception (or not) of the packets. While the transmitter always has the choice on whether to send a packet to refresh the information at the receiver or to wait to save energy, upon reception of a negative acknowledgment, there is the additional choice of retransmitting the packet in error (which has lower cost but is less beneficial to the AoI) or discarding it, and sending a new one instead.

After discussing some assumptions to make the problem tractable, the resulting model is solved via standard techniques for Markov decision problems, and numerical results are shown for some metrics of interest. This enables to derive some general criteria that can be useful in practical scenarios for wireless sensors of remote areas, especially when the available energy supply is limited.

The rest of this paper is organized as follows. Section II reviews models proposed in the literature for AoI optimization. In Section III, we outline our contribution of a Markov model where also an EHD is taken into account. The model solution is discussed in Section IV, while Section V presents numerical results. Finally, we draw the conclusions in Section VI.

2 Background

Over the last years, many studies revolved around the AoI in transmission systems, especially exploiting queueing theory in various system settings. In [2], the level of update information exchanged between nodes within a vehicle network is quantified; the AoI is essential to ensure that each node contains information on the current state of all neighbor nodes. Subsequently, in [10] and [11], an AoI-optimization approach is applied to M/M/1 systems and different techniques and queueing policies are discussed.

With the progress of technologies inherent to the Internet of things, the concept of AoI began to be applied to wireless systems. In [3] a wireless sensor network is analyzed, consisting of an energy transmitter, a sensor, and a receiver. In parallel, the research on energy harvesting made it a relevant technology in the field of autonomous systems connected wirelessly. Papers like [5, 6, 7, 12, 13, 14] apply the AoI to wireless devices equipped with an EHD.

It is also important to account for errors over the wireless channels. For data sensors, a packet may be lost and recovered via error control methodologies such as the automatic repeat request (ARQ) technique [9]. The impact of missing a data update on the AoI is considered in [15, 16, 17, 18]; in their scenarios, the channel is noisy and each packet has a constant probability p to arrive incorrectly at destination. In particular, [16] analyzes both cases with or without feedback between the transmitter and the receiver, hence the transmitter may be accordingly informed or not whether the received packet has been corrupted by the channel noise.

Our basic idea is to start from an energy harvesting system as modeled in [12], integrate it with the retransmission strategy of [16] thanks to the feedback between receiver and transmitter, and take into consideration a battery recharging system similar to that of [7], with the difference that each quantum of energy arriving from the outside recharges the battery by only one unit rather than entirely. An ARQ system for error detection [9] will be integrated into the stochastic model and we will investigate an optimal transmission policy which includes three possible choices: not to transmit, to transmit a new packet, to retransmit a packet whose previous transmission attempt is failed. We also remark that our analysis of AoI-optimal policies for ARQ systems will include some simplifying assumptions; these can be expanded by following the ideas of [18]. This last paper, that also considers AoI in ARQ systems (but does not include the EHD and its battery in the stochastic system, as we do instead), focuses on different variations of ARQ, such as including delayed feedback or multiple error probabilities for first-attempts and retransmitted packets. Also, our results can be extended to other stochastic studies of ARQ so as to include hybrid ARQ [19], correlation effects [20], or variable round-trip times [21].

3 System Model

With reference to Fig. 1, the analyzed system includes a device that receives data packets and is equipped with a rechargeable battery that discharges whenever a transmission occurs and recharges thanks to an EHD that allows it to assimilate energy from the outside. The resulting stochastic model has been implemented in Matlab [22] through a discrete-time Markov chain. We assume a fixed transmission rate and packets of identical size, which implies that the transmission time is the same for all packets, as well as their consumption of energy drawn from the battery. The system can be therefore observed in instants that are integer multiples of a fundamental quantum [4]. Thus, time is divided into slots of the same duration equal to a packet transmission time, and the energy contained in the battery can be quantized as well, so that the transmission of a packet requires exactly one unit of energy. Finally, there is no loss of generality in assuming that the energy arrives from the EHD to the battery at the beginning of the slot.

We assume that there are always packets available to transmit so as to send a fresh information update. Thus, the ARQ system works in *heavy traffic* mode [9]. The AoI at the receiver is described by a value a that is reset to 0 every time a fresh packet is received and increases by 1 at each slot. For convenience, a is limited by A_{MAX}. Except for the case of retransmissions, there is no purpose in sending outdated packets; thus, the transmitter will always send its head-of-queue packet, so as to lower a as much as possible. The battery works as a buffer to store the energy that is not immediately used, and its capacity is limited by the value E_{MAX}. We have a birth-and-death process within this energy buffer, where each arrival of energy from the outside in a given time slot recharges the battery by exactly one unit and each transmission (or retransmission) of a packet consumes one unit of energy. That is, the arrival of energy from the environment due to harvesting follows a Bernoulli distribution, with parameter η; in other words, one unit of energy arrives with probability η, or no energy arrives with probability $1-\eta$. Thus, the energy arrival rate is η, which is, incidentally, also the upper limit to the throughput. We assume that packets and energy arrivals, as well as transmissions, take place simultaneously within the time slot.

The novelty of this paper is to consider also the case of non-ideal transmission and acknowledgments send back from the receiver so as to trigger retransmissions following a negative acknowledgment, according to a standard ARQ technique. So, it is not certain that the transmissions are always successful. Failures will be considered to be independent and identically distributed, with probability p of failed transmission. Errors are always detected by the

ARQ mechanism and communicated to the device, which must decide whether to retransmit the previously erroneous packet or to transmit a new, more up-to-date packet. The state of the device is described by a triple (a, e, R) where:

- a is the AoI associated to a packet. It can assume all integer values in the range [0, A_{MAX}]
- e is the number of charge units in the battery. It can assume all integer values in the range [0, E_{MAX}]
- R is a Boolean variable, i.e., either 0 or 1, which denotes whether the previous packet was successfully transmitted or not, respectively. The case $R = 1$ offers the option to retransmit the packet or to discard it.

Thus, the total number N of states in the Markov chain is:

$$N = 2\,(A_{\text{MAX}} + 1)(E_{\text{MAX}} + 1)$$

At the beginning of each slot, the battery can charge, discharge or maintain its internal charge level. At each time slot the value of e can therefore decrease by one, increase by one or remain unchanged; the value of a can increase by one, become equal to zero or equal to one; the value of R depends on the transmission outcome. To analyze the evolution of the Markov chain, it is first necessary to establish what are the possible choices that can be made at each step, which will determine, for each starting state, the states accessible by it. We consider three choices: transmit, not transmit or retransmit a packet whose previous transmission attempt was unsuccessful. Depending on the state of the Markov chain, this constitutes the policy adopted by the system, defined as a function $y(state) = \{0, 1, 2\}$ where 0, 1 and 2 represent one of the three possible decisions:

- 0 = not to transmit
- 1 = to transmit the most recent packet (which implies to discard any pending retransmission if $R = 1$)
- 2 = to retransmit the last failed packet, which can be done only if R is equal to 1

The policies will be described through two matrices of dimensions $(A_{\text{MAX}} + 1) \times (E_{\text{MAX}} + 1)$, where the former matrix describes the policy when $R = 0$, while the latter describes the case of $R = 1$.

Starting states of the type $(a, e, 0)$ and $(a, e, 1)$ imply different policies despite the same values of a and e, especially because when $R = 1$ all three choices are available, while when $R = 0$, there is no packet to retransmit, so it is only possible to transmit a new packet or do nothing.

For the sake of limiting the parameters in the analysis, we assume that, whenever a packet is retransmitted, the probability of failure in the second transmission attempt is equal to 0 (as opposed to the value p for the first transmission). It follows that all retransmissions are successful. This is a simplification adopted in many studies of ARQ [19], justified by the underlying assumption that incremental redundancy allows for a lower error probability of retransmissions. Such an assumption can be relaxed by computing different error probabilities, but it will just lead to a cumbersome parametric analysis with the same insight.

4 Model Solution

Our purpose is to compute the steady-state probabilities of the Markov chain. To do so, we detail a constructive step-wise derivation of the individual transitions in matrix $\mathbf{T}$. Only some transitions are possible from a certain state; thus, it is useful to analyze the possible variations of each state parameter taken individually. For the battery, only two situations can occur within each time slot: either one unit of energy or no energy arrive. Then, e can: (i) increase by 1, if an energy quantum arrives and no transmission; (ii) decrease by 1, if no energy quantum arrives and there is transmission; (iii) remain unchanged, in two cases: an energy quantum arrives and there is transmission, or no energy quantum arrives and no transmission is performed.

The data in the queue influence a, which can then: (i) increase by 1, if no transmission or if the transmission is not successful; (ii) reset to 0, if the transmission is successful and the packet was the most up-to-date; or (iii) be set to 1 when $R = 1$ and a retransmission occurs.

Finally, the Boolean parameter R can: (i) remain unchanged to 0, whenever its previous value was 0 if the transmission is successful; (ii) change from 0 to 1, if a transmission is not successful, so the opportunity for retransmission (with higher success probability) arises; (iii) change from 1 to 0 whenever a retransmission occurs, since it is assumed to be always successful, or if the transmission of a new packet occurs and it is successful; (iv) remain unchanged to 1 in the case where a retransmission is available but it is decided not to exploit it and transmit a new packet instead, but its transmission is also unsuccesful.

We can summarize the possible transitions of the Markov chain according to parameters η and p as reported in Table 1 and use them together to derive $\mathbf{T}$. Also note that the full derivation of the matrix requires some boundary conditions whenever a or e are close to their maximum or minimum values. In particular, if $a = A_{\text{MAX}}$ in the next step a cannot increase, so we cap its value to A_{MAX} and analogously for e, which cannot go outside the range $[0, E_{\text{MAX}}]$.

The transitions reported by Table 1 can be collected into an $N \times N$ transition matrix $\mathbf{T}$, where a generic element t_{ij} of $\mathbf{T}$ is the one step transition probability from the i-th state to the j-th state, according to an exhaustive labeling of the triples (a, e, R). The steady-state solution of the system can be represented by π, which is a $1 \times N$ row vector. π can be derived as the solution of the fixed point condition $\pi = \pi\mathbf{T}$, combined with a normalization condition $\pi\mathbf{1}^{\text{T}} = 1$, where $\mathbf{1}$ is an all-one row vector. Vector π is useful to derive several metrics of interest, as shown in the next section.

5 Numerical Results

We evaluate the performance of the proposed policies and some terms of comparison, as the parameters that characterize the model vary. We set $A_{\text{MAX}} = 10$ and $E_{\text{MAX}} = 4$. The following metrics of interest are defined:

$$a_{\text{average}} = \mathbb{E}\left(a\right) \;=\; \sum_{i=0}^{A_{\text{MAX}}} \left[i \left(\sum_{j=(i\,(E_{\text{MAX}}+1))\,2}^{(i\,(E_{\text{MAX}}+1)+E_{\text{MAX}})\,2+1} \pi_j \right) \right]$$

Table 1 transitions of the Markov chain

starting state	→ new state	probability	description
NO TRANSMISSION			
starting state	**→ new state**	**probability**	**description**
(a,e,R)	$(a+1,e,R)$	$1-\eta$	no energy arrival
(a,e,R)	$(a+1,e+1,R)$	η	energy arrival
UNSUCCESSFUL TRANSMISSION OF NEW PACKET			
starting state	**→ new state**	**probability**	**description**
$(a,e,0)$	$(a+1,e-1,1)$	$p(1-\eta)$	no energy arrival and R=0 in starting state
$(a,e,1)$	$(a+1,e-1,1)$	$p(1-\eta)$	no energy arrival and R=1 in starting state
$(a,e,0)$	$(a+1,e,1)$	$p\eta$	energy arrival and R=0 in starting state
$(a,e,1)$	$(a+1,e,1)$	$p\eta$	energy arrival and R=1 in starting state
SUCCESSFUL TRANSMISSION OF NEW PACKET OR RETRANSMISSION			
starting state	**→ new state**	**probability**	**description**
$(a,e,0)$	$(0,e-1,0)$	$(1-p)(1-\eta)$	no energy arrival and R=0 in starting state
$(a,e,1)$	$(1,e-1,0)$	$1-\eta$	no energy arrival, R=1 in starting state and retransmission
$(a,e,1)$	$(0,e-1,0)$	$(1-p)(1-\eta)$	no energy arrival, R=1 in starting state and trasmission of new packet
$(a,e,0)$	$(0,e,0)$	$(1-p)(\eta)$	energy arrival and R=0 in starting state
$(a,e,1)$	$(1,e,0)$	η	energy arrival, R=1 in starting state and retransmission
$(a,e,1)$	$(0,e,0)$	$(1-p)(\eta)$	energy arrival, R=1 in starting state and trasmission of new packet

$$e_{\text{average}} = \mathbb{E}(e) = \sum_{i=0}^{E_{\text{MAX}}} \left[i \left(\sum_{j\in\mathscr{A}_i} (\pi_j + \pi_{j+1}) \right) \right]$$

with $\mathscr{A}_i = \{u \in \mathbb{N} : u = 2k(E_{\text{MAX}}+1)+2i, 0 \le k \le A_{\text{MAX}}\}$.

Before discussing the evolution of the model, it is interesting to describe how the optimal policies were chosen. For simplicity, the optimization was carried out by dividing the problem into three regions according to η, since we can exploit threshold effects on the energy arrival rate: namely, if the optimal policy implies to transmit for a given value of η, it will also imply so when η is greater [4, 12]. Then, for each value of η, the optimal policy was found for different values of p also satisfying the following principles:

L1. Transmission is possible only if there is energy inside the battery

L2. Whenever the AoI is low, it is best to conserve energy, especially if p is high

L3. An update is needed whenever a is high, which leads to a threshold behavior

L4. If a transmission has failed ($R = 1$), it is best to retransmit the packet whose previous transmission was not successful, rather than attempting to send a new packet, in cases where the value of η or p is high

L5. If a transmission has failed ($R = 1$), in the next step it is better to transmit a new packet, rather than retransmitting an old packet, as long as η or p are low. Note that p being close to 0 implies that retransmissions are rare and there is basically no advantage in transmitting an old packet over the most recent one since both have probability of success that is about one

L6. If in state $(a,e,\cdot)$, the decision is to transmit then, for each state $(a,e',\cdot)$ with $e' > e$ the decision to transmit must be maintained

The optimal policy can be derived through standard approaches such as value iteration (or even heuristic approaches) [4] taking into account L1-L6 and following Bellmann's optimality principle [23].

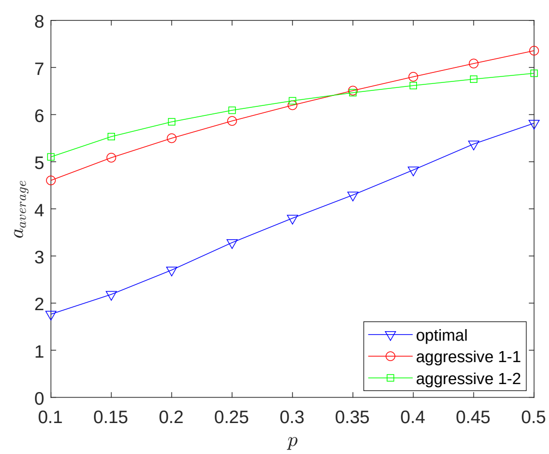

Figure 2 a_{average} vs. p when $\eta = 0.1$

We now analyze the performance based on different chosen policies and parameters. Three values of η are considered: $\eta = 0.1$, $\eta = 0.3$ and $\eta = 0.5$. The p parameter is varied between 0.1 and 0.5. For comparison, two aggressive policies are shown as benchmarks:

- Aggressive policy 1-1: always transmits when $e > 0$, and when a transmission fails, it always transmits a new fresh packet
- Aggressive policy 1-2: always transmits when $e > 0$, and when a packet fails, it attempts to retransmit it; it is guaranteed to succeed but it is less fresh

In Figs. 2–4, we evaluate a_{average} for different choices of parameter η $(0.1, 0.3, 0.5$, respectively). In particular, Fig. 2 considers a case where the energy arrival rate is low. In this case, a good energy management policy is necessary, which prevents the battery from discharging and allows the value a_{average} to stay low. Fig. 2 also shows that a_{average} with the optimal policy is much better than the aggressive policies, especially when $p < 0.3$. As η increases, Figs. 3–4 show that the gap between the optimal policy and the ag-

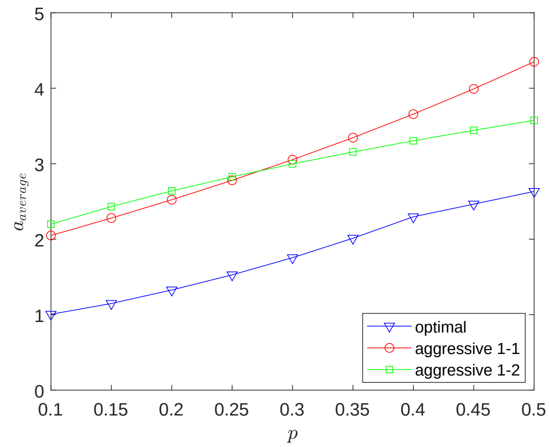

Figure 3 a_{average} vs. p policy when $\eta = 0.3$

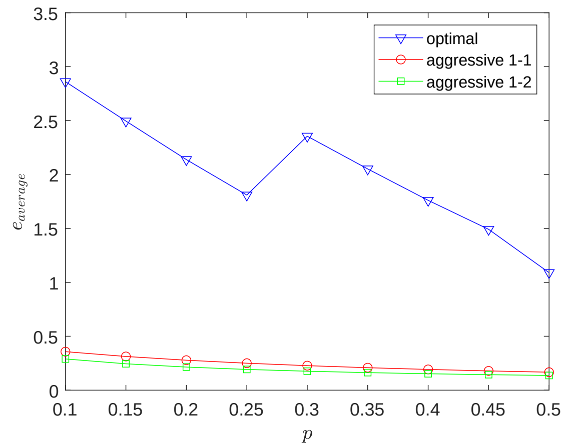

Figure 5 e_{average} vs. p policy when $\eta = 0.1$

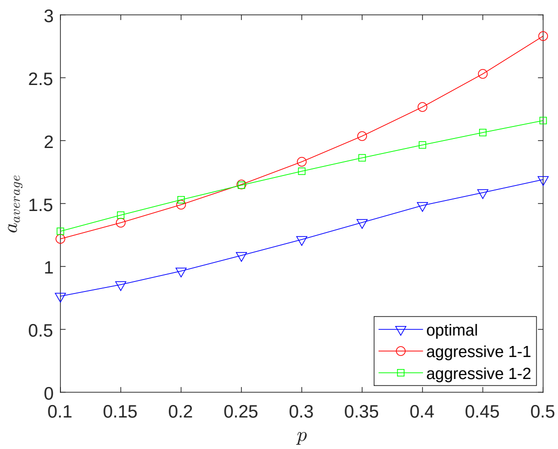

Figure 4 a_{average} vs. p policy when $\eta = 0.5$

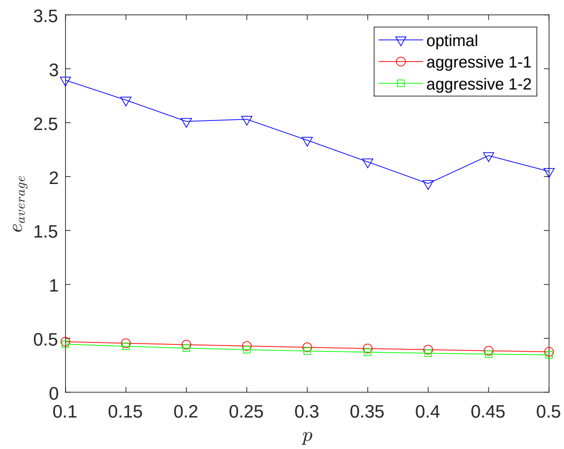

Figure 6 e_{average} vs. p policy when $\eta = 0.3$

gressive strategies reduces, because whenever energy becomes abundant, the need for transmission optimality is less stringent, and just an aggressive policy that transmits as long as there is energy in the battery would be enough.

It is eventually found that, when a transmission fails and $e > 0$, it is always better to transmit (either sending a new packet or retransmitting a previously failed one) rather than doing nothing. The value of p being low or high determines whether, when a transmission fails, it is better to transmit a new packet or retransmit the last one that failed. This also depends on the AoI, since the retransmission policy follows a threshold a^*, such that when $a \geq a^*$ a retransmission is preferable to the transmission of a fresh packet.

Figs. 5–7 show e_{average} in the cases where $\eta = 0.1$, $\eta = 0.3$ and $\eta = 0.5$, respectively. The points where the trend of the curves changes correspond to the points where the policy is changed to ensure that the AoI remains minimal. The optimal policy curve is, in all three cases, much higher than the curves relating to aggressive policies. This means that the optimal policy has a side effect of energy saving; this is justified by the optimal transmissions to happen only when really needed as opposed as whenever there is energy in the battery, which risks to deplete the battery more frequently. Notably, [4] suggests that a high e may be inconvenient as it implies that some transmission opportunities are not exploited. However, our results just show the trend for an optimized AoI (not an optimized energy consumption).

Figs. 5–7 also show some irregular trends: for example, the value increases between $p = 0.25$ and $p = 0.3$, which happens because e is not optimized (just a); thus, while the energy expenditure is generally efficient, it may not be optimized if needed to achieve a better AoI. This implies that an alternative approach, where a tradeoff is considered between AoI and energy saving, may also make sense. This can be an interesting extension to consider in future work.

6 Conclusions

A stochastic model has been developed for the study of AoI in an EHD with ARQ [9]. We considered a Markov chain tracking the battery status and packet AoI, plus an additional parameter, which takes into account the possibility of retransmitting a packet whose previous transmission failed. We derived the transmission policy minimizing the average AoI and we showed how significant gains can be obtained when the energy arrival rate is low and therefore efficient management of the battery is key. Moreover,

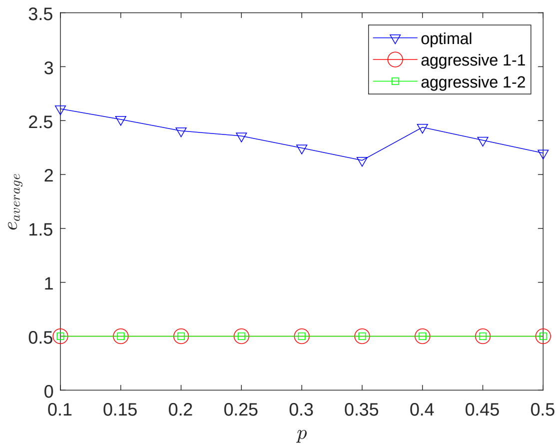

Figure 7 e_{average} vs. p policy when $\eta = 0.5$

it has been shown that, following a transmission failure, retransmission is convenient only when the success rate of transmissions is low, or the energy arrivals are frequent, or both. Otherwise, it is more convenient to discard the failed packet (even when the error probability of the retransmission is 0) and send new fresher data instead.

This study makes some assumptions that can be relaxed following other studies already available in the literature. For example, correlations in the arrival rates or the error process [20] as well as different error probabilities for retransmitted packets, according to a hybrid ARQ process [18], or a variable round-trip time [21] may be considered.

Finally, possible extensions include the trade-off between energy saving and low AoI (as opposed to always minimizing the AoI as discussed here) and the systematic derivation of simpler heuristic strategies to approximate the optimal policy with a low complexity approach.

Acknowledgment

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