Impact of Transmission Cost on Age of Information at Nash Equilibrium in Slotted ALOHA

Leonardo Badia, *Senior Member, IEEE*

***Abstract*—This letter presents an evaluation of slotted ALOHA using game theory to capture the strategic choices of the nodes, considered as independent agents that attempt to obtain updates from a shared source, with collisions preventing them from getting a usable update. Their objectives are to minimize the sum of the average age of information and a transmission cost term. The latter is an important addition to the model, shown to achieve better coordination among the nodes, so that, while the price of anarchy of the system is unbounded, a limited price of stability, approaching 1 for increasing cost, can be obtained.**

***Index Terms*—Age of Information; Game theory; Price of Anarchy; Price of Stability; Random Access; Slotted ALOHA.**

I. Introduction

With technologies for the Internet of Things (IoT) on the rise, analytical studies are increasingly considering Age of Information (AoI) [1] as a performance metric, especially for remote sensing applications. Indeed, assessing the AoI should be at least complementary, if not more valuable, to evaluations based on throughput or latency [2].

However, IoT paradigms are also generally exploiting heterogeneous uncoordinated access and distributed control. These properties relate to the usage of a random-based medium access control strategy [3] and game theory [4]. While the former is touched, even briefly, in the seminal papers posing the foundations of AoI investigations [1], the latter appears to be relatively unexplored as of now.

Inspired by this reasoning, this letter presents an analytical model for the evaluation of AoI in uncoordinated medium access protocols of slotted ALOHA type. Some adjustments are proposed with respect to [1] to precisely account for a discrete time axis. This model is subsequently framed as a game where individual players compete for accessing a shared resource, which allows them updating their information as long as they avoid collisions. The game considers AoI-based utilities for the slotted ALOHA terminals, including a transmission cost, whose role is discussed in detail.

The literature is relatively abundant with game theoretic analysis of ALOHA-like systems, but they mostly focus on maximizing throughput as the objective of the players [5]. This approach results in inefficient Nash Equilibria (NEs), as discussed in the following, where it is conversely argued that an AoI-based game can, at least in principle, achieve a better degree of coordination. There are also some game theoretic approaches to AoI, but they resort to a general representation of the medium access procedure and do not specifically focus on random access or ALOHA-like protocols. For example, in [6] the focus is on a broadcast scenario, whereas [7] addresses multiple age of information values in a distributed fashion, but the optimization is centralized. Instead, [8] takes a further step beyond and considers the nodes as distributed players driven by AoI minimization, but the medium access is not modeled specifically on slotted ALOHA as done here. A paper most similar to the present contribution is [9] where a game theoretic approach is considered with the objective of minimizing AoI, which is put in comparison with throughput, and finally random medium access is also considered with a slotted model. Yet, the analysis of that paper is entirely different because it considers the *objectives* of the networks as the players, and not the nodes, as done here. In other words, that game involves age vs. throughput as strategies that a local network can choose, and the network itself is coordinated without any strategic action within.

Moreover, in the present letter the role of transmission cost inside the utilities is discussed. On one hand, this reflects the requirement of consuming energy or activating the transmitting equipment to get updates. On the other hand, this makes sense to prevent terminals from persistently requesting updates, which is (unsurprisingly) found to be dominant strategy if no cost term is considered. Also, both the price of anarchy (PoA) and the price of stability (PoS) of the resulting slotted ALOHA system are shown, defined through the comparison of the global optimal choices versus the worst and best NEs, respectively. It is found out that, when AoI-based utilities are considered, the PoA is unbounded, which reflects the characters of total discoordination and instability of ALOHA, but the PoS can be brought down to low values as long as the cost coefficient c is high enough. Surprisingly, there is no gradual trend, but rather a precise threshold on c imposed by algebraic relationships for an additional efficient NE to appear.

II. System Model

Consider a discrete time axis divided into slots. In a slotted ALOHA system, N terminals share a resource such as a channel or a server. In the case of remote sensing, one can think of N terminals trying to get updates from a common data source. In the following, the terms "transmission" or "update" will be used interchangeably, the former being reminiscent of a random medium access where maximizing throughput is the objective, while the latter is more adequate for scenarios where the key goal is to minimize the AoI through sporadic updates.

At each time slot, the terminals independently decide whether to access the resource and perform a transmission

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L. Badia is with the University of Padova, Dept. of Information Engineering (DEI), via Gradenigo 6B, 35131 Padova, Italy. email: leonardo.badia@unipd.it

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(or update) attempt, or stay idle. Transmissions succeed only if exactly one terminal is active, while the remaining $N-1$ are idle. If more than one terminal collide, i.e., are active in the same time slot, they all fail to update. Inactive terminals are clearly not getting any update either. The AoI [1] is computed as the difference δ between the present time index and the last time slot with a successful update. Note that, differently from [1], δ is always an integer value. In particular, $\delta = 0$ if the present slot contains a successful update.

The following simplifying assumptions are made for the sake of analytical tractability. Relaxing them would only cause the model to become more involuted without any significant additional insight. First of all, a simplified ALOHA-like system is considered, where no retransmissions after a backoff are performed. This means that each terminal $j = 1, \ldots, N$ access the shared resource (i.e., the channel or the data source) with independent and identically distributed (i.i.d.) probability t_j over different time slots. This is consistent with other investigations in the field [3], [9]. While it would make sense to think of more complex strategies, e.g., where the terminals become increasingly aggressive as their AoI grows, this would cause an explosion in the game theoretical analysis of the strategic choices of each terminal, as well as their information sets and the countermoves available to their opponents [10]. Also, it would require a return channel and a general adaptive behavior that are likely outside the capabilities of commercial IoT devices, such as LoRa or Sigfox [2]. At any rate, the considerations about the PoA or PoS are unaffected, as they can still be computed over specific available strategies corresponding to the choice of a given t_j for each player j.

Moreover, it is assumed that whenever a terminal transmits alone on a given time slot, its request is always successful, i.e., failures to update are only related to collisions. Indeed, it would be easy to extend this analysis to the case of non-guaranteed decoding of the update, as per [1], by rescaling the success rate of a transmission attempt through a constant term representing the probability of decoding.

According to these assumptions, the expected value of the AoI δ_j of user j can be obtained as

$$\mathbb{E}[\delta_j](\mathbf{t}) = \frac{1}{t_j \prod_{k \neq j}(1 - t_k)} - 1 \ . \tag{1}$$

Note that (1) is an adjusted version of what derived in [1], since in that paper the evaluation of slotted ALOHA considers a continuous time variable. In the presented formulation with a discrete time axis, the expected δ_j is computed through the following steps. If $\rho = t_j \prod_{k \neq j}(1 - t_k)$ is the probability of a successful attempt by user j, and $\kappa + 1$ is a random number of slots between successful updates, whose probability is $\rho(1-\rho)^\kappa$, the expected AoI is obtained as the ratio of second and first order moments of the inter-update (discrete) time, i.e.

$$\mathbb{E}[\delta_j](\rho) = \frac{\displaystyle\sum_{\kappa=0}^{\infty} \frac{\kappa}{2}(\kappa+1)\rho(1-\rho)^\kappa}{\displaystyle\sum_{\kappa=0}^{\infty}(\kappa+1)\rho(1-\rho)^\kappa} \tag{2}$$

that, after some algebra, reduces to $\rho^{-1}-1$, as per (1).

Another aspect considered in this letter concerns the *cost* of a transmission attempt and its impact on PoA or PoS. It is assumed that each individual transmission attempt implies a fixed cost term equal to c for the terminal, paid even if the attempt turns out to be unsuccessful due to collisions. This results in a different computation of the overall utilities of the terminals and a slightly changing definition of PoA/PoS, depending on what is the overall end goal of the terminals.

In a standard slotted ALOHA analysis, the ultimate objective of the terminals would be high throughput and thus high success probability, so we can define the utility of each terminal $j = 1, \ldots, N$, as a function of the combined strategic choice $\mathbf{t} = (t_1, t_2, \ldots t_N)$ of all players

$$u_j^{(thr)}(\mathbf{t}) = t_j \prod_{k \neq j}(1 - t_k) - c\, t_j \ . \tag{3}$$

Since c often multiplies the transmission probabilities t_j, which ought to be of the same order of magnitude of $1/N$, especially to visualize the numerical results, it may be convenient to define a normalized $\widetilde{c} = c/N$. We can compute the PoA and the PoS as the ratios between the highest sum utility among all choices of $\mathbf{t}$ and the sum utility at the worst and the best NEs, respectively.

Symmetry reasons lead to the conclusion that t_j must be identical for all the terminals in the optimal choice that can be therefore written as $\mathbf{t}^* = (t^*, t^*, \ldots, t^*)$, and similarly for the NEs, aside from degenerate cases discussed later, so that if we have a number of L NEs (in mixed strategies), we can write $\mathbf{t}^{(k)} = (t^{(k)}, t^{(k)}, \ldots, t^{(k)})$ as the vector of (identical) transmission probabilities at the kth NE, $k = 1, \ldots, L$. Due to this symmetry, the PoA and PoS related to throughput can be computed as

$$PoA^{(thr)} = \frac{u_1^{(thr)}(\mathbf{t}^*)}{\min_{k \in \{1,\ldots,L\}} u_1^{(thr)}(\mathbf{t}^{(k)})} \tag{4}$$

$$PoS^{(thr)} = \frac{u_1^{(thr)}(\mathbf{t}^*)}{\max_{k \in \{1,\ldots,L\}} u_1^{(thr)}(\mathbf{t}^{(k)})} \tag{5}$$

both being larger than or equal to 1 by construction.

If instead the minimization of the expected AoI as computed through (1) is taken as the end goal, the following modifications are needed. Since it is a metric to be minimized, the expected transmission cost is to be *added* to it, rather than subtracted. The utility of terminal j, meant as a value that j prefers to maximize, can be computed with a negative sign as

$$u_j^{(AoI)}(\mathbf{t}) = -\mathbb{E}[\delta_j](\mathbf{t}) - c\, t_j = -\frac{1}{t_j \prod_{k \neq j}(1 - t_k)} + 1 - c\, t_j \tag{6}$$

and in this case the PoA and the PoS are, for consistency, the reverse ratios of (4) and (5), respectively, since the absolute values of the (negative) utilities are higher at the NE. Thus,

$$PoA^{(AoI)} = \frac{\min_{k \in \{1,\ldots,L\}} u_1^{(AoI)}(\mathbf{t}^{(k)})}{u_1^{(AoI)}(\mathbf{t}^*)} \ , \tag{7}$$

$$PoS^{(AoI)} = \frac{\max_{k \in \{1,\ldots,L\}} u_1^{(AoI)}(\mathbf{t}^{(k)})}{u_1^{(AoI)}(\mathbf{t}^*)} \ . \tag{8}$$

III. Game Theoretic Analysis

A slotted ALOHA game played by N terminals, where the utilities to maximize for each player are related to the individual throughput of the terminals, is a standard scenario often used as an introductory example [4]. It can be thought of a static game of complete information with a finite set of strategic actions, where each player has the choice between transmit (T) or stay silent (S). This game has N pure strategies NEs where one player transmits and all the others stay idle. These are clearly unfair outcomes, therefore a symmetric mixed strategy NE is considered instead.

Locally optimal values of $\mathbf{t}$ can be found as the probabilities of each player choosing action T at the mixed strategy NE, if any. Without a cost term (i.e., $c = 0$), the mixed strategy NE degenerates into choosing $t = 1$ (i.e., to always transmit), as it can be shown that T is a dominant strategy for the players. Incidentally, it is also well known that the best choice of the transmission probability vector from a global perspective would be $\mathbf{t}^*=(1/N, 1/N, \ldots, 1/N)$, which is an even simpler exercise for beginners in communications engineering.

Introducing a cost term c makes the game more interesting. In this case, $c < 1$ is required, or else S would now become a dominant strategy. Through some computations, the global optimum is found as $\mathbf{t}^* = \big((1-c)/N, (1-c)/N, \ldots, (1-c)/N\big)$, and the mixed strategy NE can be proven to be unique and given by $\mathbf{t}^{(1)} = (1-c, 1-c, \ldots, 1-c)$. Indeed, the scenario without transmission cost results as a continuous extension for $c\to 0$. The mixed strategy NE also follows the indifference criterion of game theory [10], implying that players are indifferent among any linear combination of the pure strategies in the NE support, which necessarily consists of both T and S. Thus, despite the structure of the game being more interesting due to the introduction of a transmission cost, and also the aggressiveness of the transmitters being now mitigated, the PoA (and also the PoS, due to the mixed strategy NE being unique) is always infinite, since the payoff at the NE is 0 as a consequence of the indifference criterion.

A game theoretic evaluation for AoI-based utilities becomes instead appealing. If c=0, the optimal transmission probabilities are once again found as $\mathbf{t}^* = (1/N, 1/N, \ldots, 1/N)$, which mirrors the case of the throughput-based utility [1]. It is intuitive that always attempting an update is still a dominant strategy, but this unfortunately leads to a unique bad NE, whose PoA and PoS are again infinite since the AoI explodes.

If cost $c > 0$ is introduced, the condition for the optimal transmission probability maximizing the global utility of the N players once again implies $t_1 = t_2 = \cdots = t_N = t$ but requires to solve a $(N+2)$th-degree equation

$$1 - Nt + ct^2(1-t)^N = 0\ , \tag{9}$$

and it can be observed that (9) always admits exactly one solution in the range $[0, 1]$ that can be read as a probability; it actually always falls in $[0, 1/N]$, thereby confirming the effect of the cost term to decrease the transmission probability.

From a game theoretic perspective, a NE can be derived through a one-sided maximization of the utility; that is, each player looks for a *best response* [5], [10] to the unchanged moves of the other players. Without loss of generality, focus on player 1. The NE corresponds to a point where

$$\frac{\mathrm{d}u_1(\mathbf{t})}{\mathrm{d}t_1} = \frac{1}{t_1^2\prod_{j=2}^{N}(1-t_j)} - c = 0 \tag{10}$$

and the same condition holds for any other player, replacing the indices accordingly. This leads to an interesting development, where the only choice of $\mathbf{t}$ consistent with its elements being probability values is $t_1 = t_2 = \cdots = t_N = t$, leading to a $(N+1)$th degree equation $\Theta(t) = 0$ where

$$\Theta(t) = t^2(1-t)^{N-1} - \frac{1}{c} \tag{11}$$

to find the critical point of the utilities. This is different from throughput-based utilities, where the cost term had a gradual impact in decreasing the transmission probability. For AoI-based utilities, c must be significantly high (for example, we certainly need $c > 1$, differently from the previous game) to cause a decrease in the first derivative of the utility.

In other words, for low values of c, the utility of each player is always increasing in t, which still leads to an aggressive transmission policy $t = 1$ and an inefficient NE. Yet, surprisingly, there is a threshold point γ, such that, beyond the inefficient NE $t = 1$, a better one exists if $c \geq \gamma$ with

$$\gamma = \frac{(N+1)^{N+1}}{4(N-1)^{N-1}}\ . \tag{12}$$

This can be seen from the graph of $\Theta(t)$ from (11), which has a local maximum in $2/(N+1)$ and $\Theta(t)$=0 always admits a first solution τ_1 that is negative anyways, and therefore not a legitimate probability value. If $c \geq \gamma$, the local maximum is non-negative and the function has two more zeros in $[0, 1]$ that are labeled as τ_2 and τ_3. Note that τ_2 is always between 0 and $2/(N+1)$ and gives a maximum of the utility, while τ_3 gives a minimum. Additionally, depending on N being odd or even, there may be another zero $\tau_4 > 1$, also irrelevant as a probability. As a result, in addition to the first NE (NE 1), $\mathbf{t}^{(1)}$=$(1, 1, \ldots, 1)$, a second NE (NE 2) at $\mathbf{t}^{(2)}$=$(\tau_2, \tau_2, \ldots, \tau_2)$ is obtained, which gives a bounded PoS.

To sum up, a game with AoI-based utilities including a transmission cost always gets NE 1 for $t = 1$, where utilities are infinitely low, thereby causing an unbounded PoA. This is akin to the model with throughput-based utilities, and a consequence of the distributed and unstable nature of slotted ALOHA. However, if transmission cost $c \geq \gamma$ is sufficiently high, NE 2 appears, where all terminals update with probability τ_2 that is a solution of $\Theta(t) = 0$, and is significantly less aggressive than NE 1, leading to a bounded PoS.

IV. Numerical Evaluations

This section presents some practical computations of the previously derived equations to better clarify the numerical extent of the results found. Consider a slotted ALOHA scenario, modeled as a N-player game with AoI-based utilities, where the strategic choice is the pre-determined i.i.d. probability value according to which to try to update from the source at each slot. Update attempts have cost $c = \widetilde{c}N$ regardless of whether they are successful or not, and every time an attempt

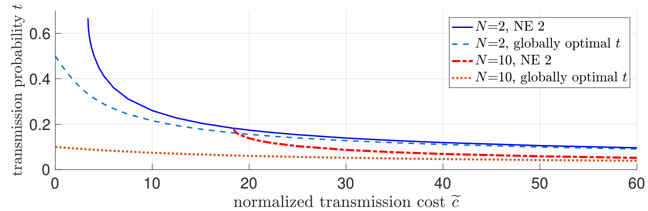

Fig. 1. Update probability t as a function of the normalized transmission cost $\widetilde{c}$, chosen at NE 2 or with a globally optimal choice.

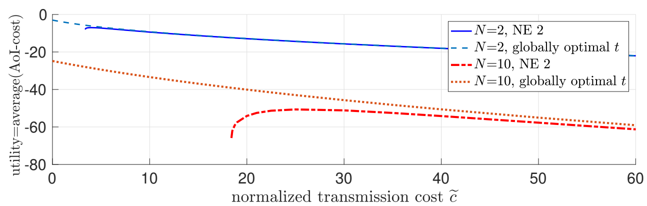

Fig. 2. AoI-based utility of each terminal, as a function of the normalized transmission cost $\widetilde{c}$, for t chosen at NE 2 or with a globally optimal choice.

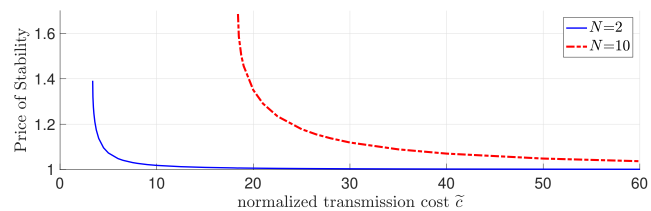

Fig. 3. Price of stability for AoI-based utility, as a function of the normalized transmission cost $\widetilde{c}$.

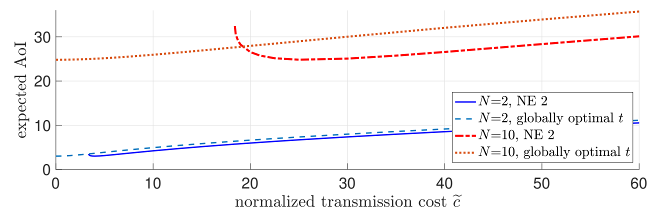

Fig. 4. Expected AoI, as a function of the normalized transmission cost $\widetilde{c}$, for t chosen at NE 2 or with a globally optimal choice.

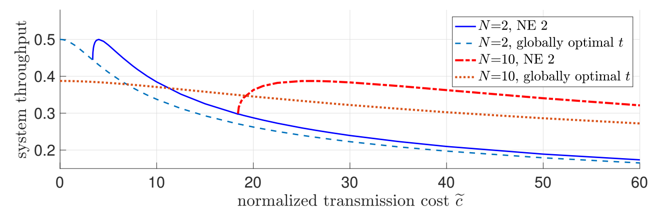

Fig. 5. System throughput, as a function of the normalized transmission cost $\widetilde{c}$, for t chosen at NE 2 or with a globally optimal choice.

does not collide with other terminals in the same time slot, it is always successful in pushing the AoI back to 0. Given the symmetry of the scenario, all approaches to solve the problem, either from a global perspective or a selfish one of a NE, result in the same update probability t for every terminal. All the results are displayed as functions of the normalized transmission cost $\widetilde{c}$, for two cases, N=2 or N=10.

Fig. 1 compares the optimal choice of the transmission probability from a global standpoint with the selfish perspective of the NE 2. Importantly, NE 1 is always present, where t=1 regardless of the cost c, whereas NE 2 only exists if $c \geq \gamma$, with γ=27/4 for N=2, which can be related to the sign of the discriminant of cubic equation (11), and $\gamma \approx 184.11$ for N=10. Remarkably, the value of $t = \tau_2$ at NE 2 starts from a much higher value than the optimal t when $c = \gamma$ but decreases rapidly. For increasing cost, the two values become closer even though, naturally, τ_2 is always greater than t^*.

The resulting utilities get even closer for increasing $\widetilde{c}$, as shown in Fig. 2 where the value of each terminal's utility is shown, once again for NE 2 with $t = \tau_2$ and the globally optimal choice $t = t^*$. Utilities are negative, and the optimal value is always higher, but the utility at NE 2 rapidly approaches it as $\widetilde{c}$ increases. For N=2 they become very similar when $\widetilde{c} > 5$ and almost undistinguishable if $\widetilde{c} > 8$. For N=10, higher values of $\widetilde{c}$ are required but the trend is similar.

This is confirmed by Fig. 3, where it is shown that, in opposition to the unbounded PoA, for a sufficiently high transmission cost, a better NE is also present and its choice gives margin to introduce some coordination, that is, the PoS becomes close to 1 with increasing $\widetilde{c}$.

Finally, Figs. 4 and 5 show the AoI and the total throughput, respectively, which for NE 2 may even get a better value than with the optimal choice of t. The reason is that the optimal value is not based on the throughput or the AoI alone, but also includes the cost; thus, NE 2 results in an overall lower utility. However, the trends of AoI and throughput at NE 2 show that, for a properly chosen cost, these metrics can be assigned to the best achievable values by an uncoordinated approach.

V. Conclusions

A game theoretic analysis of a slotted ALOHA system with AoI-based utilities including a transmission cost term was given. A closed-form derivation of the NEs was presented, showing that, while the PoA is unbounded, for sufficiently high transmission cost, an additional NE appears, which implies that the PoS can be kept limited and close to 1.

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