# Optimizing Real-Time Decision-Making in Sensor Networks

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*Abstract*—The rapid integration of digital technologies into physical systems has given rise to cyber-physical systems, where the interaction between the computational and physical components plays a crucial role. This study explores optimal decisionmaking in event detection and transmission scheduling within cyber-physical systems, emphasizing the crucial aspect of efficient decision-making. We consider the problem of monitoring and reporting about a single event taking place within a finite time window achieving a reward related to the timeliness of the status update. Thus, the objective corresponds to minimizing the age of information between the instant of the event  $x$  and the status update time  $t$ , with a further penalty for a missed event. The monitoring apparatus decides when to perform the status update without knowing the value of  $x$ , but only knowing its statistical distribution. We assume a triangular probability density function for the instant of the event taking place, with a variable average. We provide an analytical derivation of the optimal choice of the status update, highlighting interesting trends, such as the saturation in the value of  $t$  as  $x$  grows close to the limit of the observation window. This proposed problem and its analytical formalization may serve as a further foundation for the general analysis of optimal monitoring of cyber-physical systems.

*Index Terms*—Internet of Things; Age of Information; Sensor Networks; Optimal Transmission Scheduling

## I. INTRODUCTION

Real-time monitoring is becoming increasingly important in cyber-physical systems to enhance the ability of decisionmaking and achieve more efficient control [1]. This is driven by a need for immediacy of information, which is gaining momentum across various domains and applications [2], but is to be combined with the standard objectives of resource optimization [3], information security [4], and improved efficiency of the cyber-physical systems [5].

Timely reporting of measurements is generally considered crucial in highly dynamic systems, such as vehicular and transportation networks [6]. For these scenarios, the system control needs to leverage fresh and accurate information to avoid congestion, improve safety, and overall achieve customer satisfaction and/or sustainability in urban mobility [7]. To this end, several approaches have been proposed in the literature based on the general idea of minimizing the so-called Age of Information (AoI) [8]–[10]. AoI can be used to optimize the reporting from information sources towards data freshness, even for decentralized multi-agent systems [11]. Also, correlation among sensors and concurrent transmissions can improve AoI in certain scenarios [12].

However, the usual approach to minimize the average AoI suffers from potential drawbacks. First of all, it does not give guarantees on absolute freshness (i.e., it does not prevent isolated data packets from being obsolete). For this reason, some papers integrate the evaluation of the average AoI with that of peak AoI and include other more practical aspects [13], [14]. Also, most of the studies on AoI assume that continuous data exchange is required [15]. In realistic scenarios, sporadic generation of data is possible, and in the general case of context-aware communications, most applications would require timely delivery of specific events that do not continuously happen but have a specific instantiation over time. To this end, researchers have proposed metrics like Age of Task-oriented Information (AoTI) for industrial tasks [16].

This idea marks a paradigm shift concerning applications with continuous reporting, as vehicular communications that represent the original scenario for AoI-based optimization, to more general event-driven contexts, including industrial applications [17], but also other fields such as emergency or mission-critical communications [18], [19]. Also in healthcare, real-time monitoring of patient data, vital signs, and medical equipment is important but triggering an event is often only required in situations where immediate medical attention is needed [20], [21]. Similarly, when cyber-physical systems are used for the safety and security of both physical assets and individuals (e.g., in areas such as surveillance, public safety, and critical infrastructure protection, timely detection of security threats is essential for a rapid and effective response, but only to specific events [22]). In all these contexts, timely reporting of events allows for the immediate detection of criticalities and anomalies, and the ability to respond promptly is crucial for preventing or mitigating potential risks. However, the general effort to keep a low average AoI may be inadequate as it leads to considerable effort and resource wastage, whereas only sporadic events need to be monitored [23].

In this work, we propose the analysis of optimally tracking an isolated event happening at time  $x$  over a finite time window [24]. Such an event may represent a system change or a critical episode, but its precise instant  $x$ , while bound to fall within the window, is not known and only statistically characterized through its pdf. The delayed reporting of the event at time  $t$ incurs a linear penalty in line with the definition of AoI [9]. In the case of too early monitoring for the event, the total penalty until the end of the window is paid. For the sake of analytical tractability, our investigation focuses on a triangular probability distribution function (pdf), with variable mode (i.e., peak value) denoted as a, which allows us to compute the objective function in closed form but makes it possible to skew the distribution of the event  $x$  toward a specific instant inside the observation window. The problem becomes a single variable optimization for  $t$  as a function of  $a$ , which is also numerically evaluated and discussed.

We argue that this preliminary study can be generalized to multiple events and monitoring instances following the same analytical pattern [25]. Our approach not only contributes to theoretical foundations but also provides practical insights into optimizing decision points and understanding temporal dynamics. Our study demonstrates that, as the mode of probability distribution increases, optimal penalties decrease, and corresponding transmission times exhibit specific trends. These trends reach saturation as the pdf of the monitored event becomes increasingly skewed towards the end of the observation period. Further research can contribute to finding intelligent decision-making guiding event detection and transmission scheduling strategies in dynamic and interconnected environments [26].

The rest of this paper is organized as follows. In Section III, we present the system model. Section IV develops the analysis and finds the closed-form solution to the optimization problem. We present numerical results in Section V and we conclude in Section VI.

# II. RELATED WORK

The concept of Age of Information (AoI) as a metric for optimizing information reporting from various sources has been the subject of a sizable body of research. The age metric proposed in [8] accounts for the constrained network resources and gauges how quickly the recipients receive status updates. It offers information on how frequently status updates ought to be produced in order to reduce aging and guarantee ontime delivery. The authors of [9] aim to optimize information reporting by taking into account the age of updates and creating effective techniques to minimize delay and maximize freshness by utilizing the concept of AoI. In a different study, the concepts of AoI and game theory are used to optimize information reporting by determining the optimal update plan for the strategic sources while taking into account the sources' individual costs as well as the global objective of minimizing AoI [10].

In the context of decentralized multi-agent systems, the AoI metric has received a lot of attention [11]. In order to guarantee complete data freshness, several researchers have expanded their investigations to include peak AoI and other practical factors, even though the majority of studies have focused on decreasing the average AoI. [13] investigates the likelihood of peak-age violation and delay in a point-topoint communication system using brief information packets. The study offers insights into the system parameters that impact data freshness, such as frame size and undetected error probability, by examining the delay violation and peak-age violation probabilities. The authors in [14] examine synchronization strategies that lower the overall AoI and maintain accurate timestamps. Additionally, they look into how different transport layer protocols and congestion control mechanisms contribute to minimizing the AoI and maximizing data freshness.

However, in situations where data generation is sporadic or communication is event-driven, conventional methods that minimize average AoI might not be adequate [15]. To address the specific needs of industrial tasks, researchers have suggested alternative metrics like the Age of Task-oriented Information (AoTI) in response to this limitation. By taking into account the amount of time that has passed between the generation of the first sampling data of the newest task and the last successfully received system update at the receiver, the Age of Task-oriented Information (AoTI) metric in industrial wireless sensor networks accurately measures the freshness of system information [16]. For a given task, the AoTI metric considers the processing of data from multiple multi-type sensors. AoTI gives an accurate indication of how recent the system information is in sensor networks by timing the interval between the creation of the sampling data and the receipt of the system update. Because of this paradigm shift, AoIbased optimization is now applied in a wider range of eventdriven contexts, such as industrial applications, emergency communications, and healthcare, rather than just continuous reporting scenarios like vehicular communications [17]–[21].

Real-time monitoring in healthcare environments, in particular, has brought attention to the significance of only triggering events when immediate attention is needed [20], [21]. Similar to this, timely detection of particular events is essential for efficient responses in areas like critical infrastructure protection, public safety, and surveillance in cyber-physical systems used for safety and security reasons [22]. These various contexts highlight how important it is to report anomalies and criticalities immediately in order to detect them right away.

## III. SYSTEM MODEL

We consider a problem of timely event detection and reporting in the context of an observation window of normalized duration  $L = 1$ , for a cyber-physical system consisting of an event source generating a relevant phenomenon (such as an alert or a critical system condition) and a transmitter/receiver pair to describe the update sent about it to a remote observer [23]. We denote the timing of the event as  $x$  and the transmission instant as  $t$ . For the specific analysis presented in this paper, the communication part is assumed to be fully reliable, even though an extension with channel erasures and/or delays would be immediate along the lines of [18], [25], [27].

The transmission of the status update is guided by the general criterion that  $t$  ought to be bigger than  $x$  but as close to it as possible [17], [22]. We distinguish between two cases: (i) the status reporting is performed after the event, i.e.,  $t > x$ . In this case, the event is detected at time t. Conversely, the case where  $t < x$  corresponds to a missed



Fig. 1. Instantaneous penalty depending on the reporting time if  $t < x$ .



Fig. 2. Instantaneous penalty depending on the reporting time if  $t > x$ .

detection of the event, and the event is only noted at the end of the transmission window. Thus, the detection happens at  $D(t, x) = t + (1-t)1(x-t)$ , where  $1(\cdot)$  is a unit step function.

Accordingly, if we consider a generic  $u$  inside the window, we can set a penalty equal to the age of information (AoI) of the event reporting [9]. The instantaneous value of this penalty  $p_{t,x}(u)$  at time u as being 0 before the event happens, then growing linearly until it is finally detected, which happens either at time t (if  $t > u$ ) or at the end of the window. Thus,  $p_{t,x}(u)$  can be expressed as (1).

$$
p_{t,x}(u) = \begin{cases} u - x, & \text{if } x < u < D(t, x) \\ 0, & \text{otherwise} \end{cases}
$$
 (1)

This linear increase of the penalty propagates over time, and the objective of the optimization is the integral of the penalty over the entire window, i.e. (2).

$$
\mathcal{P}(t,x) = \int_0^L p_{t,x}(u) du = \begin{cases} (1-x)^2/2, & \text{if } t < x \\ (t-x)^2/2, & \text{if } t > x \end{cases}
$$
 (2)

This total penalty  $\mathcal{P}(t, x)$  actually can also be seen as an average penalty over the time window, since we considered a normalized time interval  $L = 1$ .

Illustrated in Fig. 1 and 2 are the two options discussed previously. If the event monitoring is scheduled at time  $t$ , depending on the occurrence of the event, we can distinguish between the case  $x = x_1 < t$  and  $x = x_2 > t$ , leading to a different value of the total penalty  $P(t, x)$  as per (2).



Fig. 3. Probability density function of event  $x$  inside the window.

For what concerns the actual position of  $x$ , we assume it is randomly distributed within the horizon and we adopt a triangular shape for its pdf, to reflect a possible uneven distribution of the event. In particular, we use a parameter a to denote the mode, i.e., the peak of the distribution. This implies that the pdf governing the occurrence of the event  $x$ , denoted as  $f(x)$ , is set as (3).

$$
f(x) = \begin{cases} \frac{2x}{a}, & \text{if } 0 \le x \le a \\ \frac{-2x+2}{1-a}, & \text{if } a < x \le 1 \\ 0, & \text{otherwise} \end{cases}
$$
 (3)

The triangular shape of the pdf signifies that the probability of the event varies on either side of the mode, with a diminishing likelihood as we move away from this central point. A graphical depiction is reported in Fig. 3.

# IV. ANALYTICAL FRAMEWORK

Building upon the previously defined system model, we can underpin the decision-making process for an advance scheduling of the observation event  $t$  following the objective of minimizing the expectation of the total penalty  $\mathcal{P}(t, x)$ , which we denote as  $P(t) = \mathbb{E}_x[\mathcal{P}(t,x)]$ , where  $\mathbb{E}_x[\cdot]$  is the expectation over  $x$ . Thus, from  $(2)$  and  $(3)$ , the expected total penalty can be expressed as (4).

$$
P(t) = \int_0^t \frac{(t-x)^2}{2} f(x) \, dx + \int_t^1 \frac{(1-x)^2}{2} f(x) \, dx \tag{4}
$$

The resulting piece-wise expression for  $P(t)$  is (5).

$$
\begin{cases}\n\int_0^t \frac{(t-x)^2}{2} \frac{2x}{a} dx + \int_t^a \frac{(1-x)^2}{2} \frac{2x}{a} dx \\
+\int_a^1 \frac{(1-x)^2}{2} \frac{-2x+2}{1-a} dx, & \text{if } t \le a \\
\int_0^a \frac{(t-x)^2}{2} \frac{2x}{a} dx + \int_a^t \frac{(t-x)^2}{2} \frac{-2x+2}{1-a} dx \\
+\int_t^1 \frac{(1-x)^2}{2} \frac{-2x+2}{1-a} dx, & \text{if } t \ge a,\n\end{cases}
$$

By solving the integrals,  $P(t)$  can be expressed as (6).

$$
\begin{cases}\n\frac{-2t^4 + 8t^3 - 6t^2 + a^3 - 3a^2 + 3a}{12a}, & \text{if } t \le a \\
\frac{-2t^4 + 8t^3 + 6at^2 - 18t^2 - 4a^2t + 12t + a^3 - 3}{12(a-1)}, & \text{if } t \ge a\n\end{cases}
$$
\n(6)

This characterizes the AoI-related dependence on timely event detection.

These closed-form formulations provide a comprehensive framework for evaluating and optimizing the penalty associated with event detection and transmission scheduling, considering the dynamic interplay between the event pdf, the transmission timing, and the resulting penalty [24].

The impact of the system parameters (in our case, the mode of the event distribution  $a$ ) and the control choice  $t$ can therefore be evaluated by solving an optimization problem formalized as (7).

$$
\min_{t}, \quad P(t) \n\text{s.t.,} \quad 0 \le t \le 1
$$
\n<sup>(7)</sup>

We remark that this simple formulation can be generalized to more complex contexts, i.e., including a different pdf than (3) or multiple monitoring instants [25].

# V. RESULTS

In this section, we explore the relationships between the value of the penalty  $P(t)$ , the chosen transmission time t, the mode of the probability distribution function  $a$ , and the resulting optimized system control.

The graph in Fig. 4 shows the direct connection between penalty  $P(t)$  and transmission time t, following from (6) and showcasing distinct curves for varying values of the mode  $a$ of the probability distribution of  $x$ . This visual exploration provides insight into how the timing of status monitoring and the statistics of the event influence the penalty function. In particular, it is sensible to consider the optimal value of the penalty  $P^* = \min_t P(t)$ , as a function of a, and the corresponding minimizing value of  $t$ , denoted as  $t^*$ , for further discussion in the following.

These results highlight a noteworthy trend, namely, as the mode  $a$  of the pdf increases, the optimal penalty  $P^*$  decreases, and the minimizing point  $t^*$  grows. To better understand this behavior, Fig. 5 considers  $P^*$  as a function of a. The



Fig. 4. Relationship between t and the achieved penalty  $P(t)$  for different values of a.



Fig. 5. Optimal penalty  $P^*$  vs. the mode  $\alpha$  of the pdf of the event.

behavior is monotonically decreasing, with a notable change of convexity when  $a = 0.634$ . This is a consequence of the instant of the event being bound within the horizon, and the detection is ultimately constrained to the last instant of the window even if  $t < x$  (missed event) so that the instantaneous penalty  $p_{t,x}(u)$  stops growing after 1 as per (1). In other words, we consider that the event under monitoring is always reported at the end of the window (i.e., in the case of missed detection, we have a detection at  $D(t, x) = 1$ .

Furthermore, the optimized system can calibrate the reporting instant  $t$  in the intermediate values of  $a$ . This is a remarkable outcome considering that the span of the penalty is overall two-fold, for  $0.2 \le a \le 0.8$  the difference is just within a 30% range.

The flex point of the curve around  $a = 0.634$ , which

corresponds to a minimal penalty  $P^* = 0.0334$ , also suggests a change of behavior in the optimization. If we further investigate the penalty-minimizing transmission time  $(t^*)$  versus  $a$ , which is addressed in Fig. 6, we see that the initial behavior of  $t^*$  is linearly increasing, but with a relatively low slope since even  $a = 0$  attains the lowest value of  $t^* \approx 0.446$ . For larger values of  $a$ ,  $t^*$  increases, still always being around the middle of the observation window, thereby suggesting that as a grows, i.e., the event is more likely to happen toward the end of the window, the optimal reporting favors a more protracted approach. However, when the point  $a = 0.634$  is reached,  $t^*$  saturates (also to this very value, i.e.,  $t^* = 0.634$ ), which means that it is preferable to keep the observation point  $t$  at that value. This is because the penalty function is not growing beyond the end of the finite horizon. Thus, it is more convenient to try to report the event at that specific intermediate point of the window rather than postpone  $t$  and risk an increased penalty if  $x < a$ .



Fig. 6. Optimal transmission time  $t^*$  vs. the mode  $a$  of the pdf of the event.

#### VI. CONCLUSIONS

Real-time decision-making driven by AoI within the context of cyber-physical systems follows intricate dynamics that bear theoretical insights and practical implications [8]. The integration of timely event detection and transmission scheduling is paramount in obtaining control efficiency and achieving the objectives of the next-generation communication systems for time-sensitive applications [3].

We considered a scenario of event detection over a finite horizon, which is only known through the prior statistics. This means that we decide the monitoring instant  $t$  only knowing the pdf of the event location  $x$ , aiming at the minimization of an AoI-related penalty that distinguishes between the cases of correct or missing detection of the event. We further performed a numerical analysis for the case of  $x$  following a triangular pdf, whose peak  $a$  (the statistical mode) is taken as a variable parameter.

Building upon this foundation, we formulated a mathematical optimization to quantify penalties associated with late event detection and missed transmissions [16]. The analytical framework offers actionable insights for decision optimization, aligning strategies with the temporal patterns suggested by the probabilistic nature of event occurrences.

The insights gained from this research can prompt the development of intelligent systems that are not only efficient but also adaptable to the probabilistic nature of events in dynamic environments. Moreover, the study lays the foundation for extending research into the realm of probability distribution functions, offering a promising avenue for extended investigation. Future work can experiment with specific applications and/or various probability distribution functions, to extend the analysis to different kinds of uncertainties inherent in dynamic environments.

At the same time, the interaction of multiple intelligent agents taking subsequent actions can be envisioned, both in a dynamic setup [26] and/or from a game theoretic perspective [10], to investigate the consequences and the resulting efficiency of strategic decision control.

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