Analysis of Age of Information Under SR ARQ

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***Abstract*—An analytical derivation of age of information is provided for a data stream with sequential content transmitted over an error-prone channel, using a selective repeat automatic repeat request for error control. The impact of the error process and/or the round trip time on the resulting age of information is assessed. The analysis allows to generate the entire statistics of AoI. The results highlight consequences such as a mild correlation in the error process, where the error bursts are not significantly longer than the round trip time, causing a decrease in AoI. This can serve as a reference to devise retransmission-based error control that is efficient in terms of information freshness.**

***Index Terms*—Age of Information; Queuing analysis; Automatic repeat request; Markov processes, Error analysis.**

I. Introduction

In the recent literature, a trend has emerged to quantify freshness of status updates between a remote transmitter and a receiver through age of information (AoI) [1]. These investigations give an analytical characterization reminiscent of classic queueing systems, as the moments of AoI can be connected to delay terms of the queue [2]–[4].

Several studies employ AoI as a performance metric akin to throughput or delay at various levels of the protocol stack, such as the data link [5] or the transport layer [6]. Most investigations, however, consider a data flow where independent packets carry atomic information. In reality, multimedia data streams such as video flows [7], or coming from real-time sensor measurements [8] often apply differential encoding for data compression and/or require a temporal consistent reading to gain proper context. As a result, the streams need a *sequential* interpretation, in that they must be read in a precise order.

In these cases, it is required to resequence packets that went out of order [9]. Moreover, the reception of a recent packet does not improve the data freshness at the receiver's side until all previous data are resequenced. If intermediate packets are lost due to channel erasures, it makes sense to provide error control through automatic retransmission request (ARQ) [10].

Such a problem was acknowledged already by classic studies [11]–[13] where the focus was, for historical reasons, on delay rather than AoI. The standard taxonomy of ARQ considers stop-and-wait, go-back-N, and selective repeat (SR) ARQ, the last being the most performing implementation but also the most demanding in terms of features at the receiver's side (a resequencing buffer is required to reorder packets received out-of-sequence) as well as the most complex to analyze. In some cases, a simpler version called *ideal SR* ARQ is considered [14], where the round trip time is set to 0, in which case all three ARQ schemes coincide.

Many AoI papers [15]–[18] take the simpler perspective of ideal SR ARQ, ignoring the round trip time. The focus is often on whether to implement ARQ or not, instead of quantifying its AoI performance. The contribution of this letter is to bridge classic SR ARQ studies with an analytical derivation of AoI of sequential content transmitted over an error-prone channel, whose erasures are counteracted by SR ARQ.

The model considers a source that always has fresh information available to send to a receiver. The data flow is split into packets and the transmission medium is a discrete time Markov channel [7], with time slots equal to the packet transmission time. Correlation among channel states [19] is considered to evaluate its impact on AoI. The receiver asks for retransmission of packets in error, by sending a negative acknowledgment over an out-of-band feedback channel. Packets are sequentially encoded, thus their in-order delivery is required; the receiver must re-sequence them, as per the standard SR ARQ assumptions. The round trip time between the source and the receiver is non-zero, which marks a difference with most of the related literature. These aspects are also considered in other papers [2], [9], but rarely together.

The first contribution given here is the definition of AoI for such a system. Then, AoI is statistically characterized with a matrix-geometric approach [11], [20]. This consists of framing the system state in a larger Markov chain that is solved for the steady-state probabilities, which derives the AoI statistics.

Finally, numerical evaluations are discussed to highlight interesting trends and hinting at useful properties to improve the system performance in terms of AoI, especially related to correlation. Remarkably, this is made possible by the matrix-geometric approach and such considerations would go overlooked in simpler investigations just focusing on independent and identically distributed (i.i.d) error processes.

II. Background

Data streaming creates a flow of information where the order of packets is critical for proper processing at the receiver's side. This includes multimedia streams, content download, real-time measurements from IoT sensors, ultra-low latency communications, and many other applications [7], [21] expected for next generation systems. The essence of using TCP at the transport layer is to provide reliable in-order delivery of content by means of selective retransmissions [6].

Timeliness of data can be captured through AoI, defined [2] at time t as the quantity $\delta(t) = t - \sigma(t)$, i.e., the time interval

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elapsed since the generation instant $\sigma(t)$ of the last decoded information. Now, denote the instants of data generation as τ_i, $i \in \mathbb{Z}$, and associate them with binary values $r_i(t)$ where $r_i(t) = 1$ if the content generated at time τ_i was correctly received at time t, 0 otherwise. For atomic contents without any interdependencies, AoI would be computed considering $\sigma(t) = \max\{\tau_i : r_i(t) = 1\}$. But this is not appropriate for data streams with sequential content, where in-order delivery requires that a packet is decoded not just when it is correctly received, but rather, all previously generated packets are as well. Thus, the definition of AoI follows from imposing

$$\sigma(t) = \tau_x \,, \qquad \text{where } x = \arg\min_i \{r_i(t) = 0\} - 1 \,, \quad (1)$$

that is, the generation instant of the last of the earliest unbroken sequence of correctly received packets at time t.

This computation prompts for a reliable reception of all packets; thus, it makes sense to consider the application of ARQ for error control. The analysis of AoI can be connected to the study of queueing delay in ARQ systems, which dates back to historical references such as [10], where packets follow a Poisson arrival process and are subject to i.i.d errors. In [9], the impact of correlation on the queue was further investigated. The delivery delay was studied in [22] for its impact on higher layers. Feedback errors are studied in [23] and variable round trip times in [24]. Other comprehensive studies [11], [13] of delay terms for a sequential content exploit a matrix-geometric approach. However, none of these studies tackle AoI, which was made popular only by recent influential studies such as [1], [2], and expanded to follow queueing system evaluations [3], [21]. Notably, some authors are to date proposing Markov models and matrix-geometric approaches to this end [20].

There are also attempts at studying AoI in ARQ systems, under different kinds of retransmission techniques. For example, [16] studies freshness of information for different ARQ models (plain ARQ and hybrid ARQ of types I and II), but the implementation follows an ideal ARQ model [9], i.e., the impact of the round trip time is not considered. Analogously, [4] considers a queue with hybrid ARQ, but again the retransmission model does not keep into account a round trip time and resequencing of packets.

An interesting tradeoff often explored in studies about AoI for (hybrid) ARQ systems is between energy consumption and information freshness [17], [25], sometimes with energy harvesting. The underlying assumption is that incremental redundancy of packets makes it possible to choose between retransmission, which is less fresh but has improved reception, or sending newly generated data [1]. In the present study, we assume the content to be sequential, hence packets cannot be discarded, but are instead selectively retransmitted until the whole line of transmitted packets can be resequenced, which may cause AoI to grow in the meantime.

These research lines are further explored in [26] for multicast, or in [15] and [27] for two-hop networks, but with the same reasoning about the convenience of attempting retransmissions versus generating a new packet, so once again the assumption is that a failed transmission can be arbitrarily replaced, which would contrast with the assumptions made here of non-zero round trip time and sequential content.

Finally, the closest reference to the present letter is [18], which indeed considers sequential content and the impact of round trip delay. However, the two kinds of ARQ analyzed are stop-and-wait and go-back-N (called reactive and proactive ARQ in the paper). The third classic scheme, SR ARQ, is not considered, despite being the superior and ultimate choice, if a resequencing buffer is available. This letter completes the contribution with SR ARQ, which is the most complex case.

Also, its study can serve as a reference for further improvements of the error control mechanism toward fresh status updates for real-time content. Indeed, the recent literature has seen some interesting AoI-aware modifications of existing classic transmission schemes at different layers of the protocol stack, such as age-threshold slotted ALOHA [28] or the age control transport protocols ACP and ACP+ [6]. In this very spirit, the present contribution can represent a foundation to investigate age-aware retransmission mechanisms.

III. Analysis

Take a source able to continuously send status updates to a receiver through a slotted noisy channel. In ARQ studies, this is said to be a source in *Heavy Traffic* [14], while in the AoI literature this is called a *generate-at-will* model for the updates [1]. Besides being sensible for the envisioned applications of real-time multimedia flows or remote sensing, this scenario highlights the effect of retransmissions that must break the persistency of the data flow. The packet transmission time corresponds to one slot, and channel errors are erasures, so that a packet in error is simply lost. Under a stringent timeout and error-free feedback, erasures are detected at the receiver's side, and a negative acknowledgment is sent back, which triggers a retransmission. Feedback errors can be treated as just causing an increase in the channel error rate [23]. Retransmissions occur after a round trip time, set to m slots, from the previous transmission attempt. They are prioritized, i.e., the flow of status updates is paused to accommodate them, since erasures block the release of all following packets, even those that are correctly received. As a side note, as customary in SR ARQ investigations, delay terms, and therefore also AoI, are computed at the transmitter's side, neglecting the propagation delay. This can be treated as a constant bias approximately equal to $m/2$, which does not affect the shape of the statistics.

If the source only sporadically sends data [13], one must distinguish whether this happens because of limited transmission opportunities, or lack of fresh information at the source's side. In this latter case, one ought to check how retransmissions can be interleaved with regular transmissions. To achieve timely status updates, in the case of sporadic generation, the source can send more replicas of the same data to proactively counteract the erasures [29]. This can be an interesting extension that is left for future work.

The data link layer aims at full reliability, thus packets kept getting retransmitted if needed, until correct reception, and the buffer size at the receiver's side is unlimited, i.e., no overflow is possible. While these are standard assumptions, they can be relaxed with just minor variations in the model, e.g., by capping the number of retransmissions or pending packets [7] – in reality, this simplifies the resulting Markov model.

The channel is represented as a two-state discrete time Markov chain, where 0 and 1 denote the error-free and the erroneous states, respectively. Despite its simplicity, this model (easily extendable to larger chains if needed) is shown in the literature to be adequate to account for error correlation in many cases of interest [14]. The channel transition matrix is denoted as $\mathbf{P} = \{p_{ij}\}$, $i, j \in \{0, 1\}$, resulting in a steady-state channel error probability $\varepsilon = p_{01}/(p_{01} + p_{10})$ and an average error burst length $B = 1/p_{10}$. Errors are i.i.d if $B = (1-\varepsilon)^{-1}$.

Following a matrix-geometric approach, a larger *ARQ chain* is defined, whose state tracks the memory of the entire system. Due to the cyclic nature of the SR ARQ, where m packets are pending at each time, the state must be an m-dimensional vector $(s_0, s_1, \dots, s_{m-1})$. The convention adopted is that s_0 refers to the last transmitted packet in the current time slot, whereas s_j, $j > 0$ denotes a packet transmitted j slots ago. As per the values of the elements $\{s_i\}_i$ of this vector, they ought to track the number of failed transmission attempts (or equivalently, the retransmissions) that each packet underwent. Thus, $s_i = 0$ denotes that the ith past slot in the timeline is occupied by a packet that was immediately received without errors at its first transmission attempt.

Along this line, $s_i = 1$ denotes a slot occupied by a packet that failed its first transmission attempt. This packet is still being retransmitted and the number of failed transmission attempts can grow further. Thus, $s_i = k$, $k > 0$ denotes that, as of now, the ith packet attempted transmission k times without success. This also means that the channel state during its transmission was 1, which will track the channel state evolution. Hence, a conventional representation is needed to denote that the packet experienced $k > 0$ erroneous transmissions, and then another one (the last) that was eventually successful. To this end, negative values are employed, so that the value corresponding to this case is $-k$. This keeps consistency with the 0th case, and for an ith entry whose value is s_i, the channel state can be computed as $\big(s_i + |s_i|\big)/2$.

It is immediate that the state transitions of vector $\mathbf{s}(t) = \big(s_0(t), s_1(t), \dots s_{m-1}(t)\big)$ are Markov. Indeed, the last entry also contains the current channel state and thus track the Markov channel evolution, and it evolves depending on whether the channel transits to error-free or one of the erasure states as well as the value of the *last* entry in the vector. If the channel is error-free, the first entry evolves at time t+1 to $-s_{m-1}(t) - |s_{m-1}(t)|$, otherwise it evolves to $\big(s_{m-1}(t)+|s_{m-1}(t)|\big)+1$. Finally, all other entry of the vector state just shift by one position, i.e., $s_k(t+1) = s_{k-1}(t)$ for $0 < k < m$. A graphical summary of these transitions, for the case $m = 3$, is reported in Fig. 1.

The transitions can be collected into a matrix $\mathbf{T}$ that is sparse as each state only has 2 outgoing transitions, depending on the channel evolution toward either error-free state or erasure. In principle, there are infinitely many states $\mathbf{s}$ and it

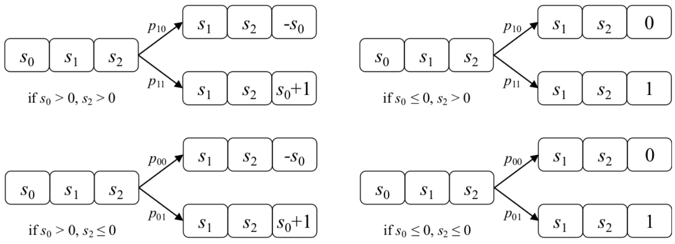

Fig. 1. State transitions of the Markov chain for $m = 3$.

can be convenient, as already discussed, to set an upper limit F on the value for the allowed retransmissions in the s_i entries, so large that it results in a negligible approximation. In this way, the number of possible states becomes $N = (2F + 1)^m$ and the Markov model can be solved for the N-sized steady-state probability vector $\boldsymbol{\pi}$ of the ARQ chain that satisfies $\mathbf{T}\boldsymbol{\pi} = \boldsymbol{\pi}$ and $\sum_{\mathbf{s}} \pi(\mathbf{s})$=1 with the following matrix equation

$$\boldsymbol{\pi} = \left[\begin{array}{c} \mathbf{T} - \mathbf{I}_N \\ \mathbf{1}_N \end{array}\right]^{-1} \left[\begin{array}{c} \mathbf{0}_N \\ 1 \end{array}\right], \qquad (2)$$

where $\mathbf{I}_N$ is the identity matrix of size N, $\mathbf{1}_N$ is a row vector of all 1s, $\mathbf{0}_N$ is a column vector of all 0s.

Although N grows exponentially in m, the system is overall tractable due to the sparsity of $\mathbf{T}$. Alternatively, the $\pi(\mathbf{s})$s can be computed through the following balance equations

$$\begin{aligned}
\pi(0, s_1, \dots, s_{m-1}) &= \textstyle\sum_{k\leq 0} \pi(s_1, \dots, s_{m-1}, k)\, p_{\xi 0} && (3)\\
\pi(1, s_1, \dots, s_{m-1}) &= \textstyle\sum_{k\leq 0} \pi(s_1, \dots, s_{m-1}, k)\, p_{\xi 1} &&\\
\pi(k, s_1, \dots, s_{m-1}) &= \pi(s_1, \dots, s_{m-1}, k-1)\, p_{\xi 1} \quad \text{if } k \geq 2 &&\\
\pi(k, s_1, \dots, s_{m-1}) &= \pi(s_1, \dots, s_{m-1}, -k)\, p_{\xi 0} \quad \text{if } k \leq -1 &&\\
&\text{where } \xi = \big(s_1 + |s_1|\big)/2\,. &&
\end{aligned}$$

Once the $\pi(\mathbf{s})$ probabilities have been computed, AoI of every state $\mathbf{s}$, denoted as $\delta(\mathbf{s})$, must be determined, which is not trivial. To ease the understanding of the computations, some examples are reported in Table I for the case m=3, which is not restrictive and can be easily generalized.

The computations can be understood through these remarks:
1. if $\mathbf{s} = \mathbf{0}_m$ then AoI is 0.
2. if $\mathbf{s}$ contains at least one positive entry, the highest of them represents the oldest packet still pending acknowledgment. If two or more entries have the same highest value, consider the one with highest index, since it represents the oldest of them. Denote its index as j and its value as v. Then, AoI of this state can be written as $m(v-1) + j + 1$.

TABLE I
EXAMPLES OF AOI COMPUTED FOR $m = 3$.

$\mathbf{s}$=(s_0, s_1, s_2)	corresponding AoI $\delta(\mathbf{s})$
$(1, 0, 0)$	1: missing packet in s_0, sent now
$(0, 0, 1)$	3: missing packet in s_2, sent 2 slots ago
$(0, 0, 2)$	6: now packet in s_2 was sent 5 slots ago
$(-1, 2, 1)$	5: blocking packet is actually in s_1
$(-1, 0, -2)$	1: all correct packets, most recent is 1 slot old
$(-2, -3, -1)$	5: all correct packets, most recent is 5 slots old

3. if all the entries in s are non-positive, there are no packets pending acknowledgment. AoI is not necessarily 0, as it depends on when the most recent packet was transmitted. This is denoted by the maximum value within s (since they are all non-positive, it is the one that is lowest in absolute value). In case of two or more entries with the same maximal value, it is the one with lowest index. Denoting the index as j and the value as v, AoI is $(-v)m+j$. Notably, if s_0=0 and all other values are non-positive, then AoI in the current state is 0 as all previous packets are resolved and the last successful packet keeps the receiver with the freshest information.

Formalizing what above, $\delta(\mathbf{s})$ is computed from s through

$$\begin{aligned} v &= \max(\mathbf{s}) \qquad (4)\\ j_0 &= \min\left(\{j : s_j = v\}\right)\\ j_1 &= \max\left(\{j : s_j = v\}\right)\\ \delta(\mathbf{s}) &= \begin{cases} m(v-1)+j_1+1 & \text{if } v>0\\ j_0 - mv & \text{if } v \leq 0 \end{cases} \end{aligned}$$

The analysis above allows for the closed-form derivation of all the relevant metrics of interest. The average AoI, on which we will focus for the performance evaluation of the next section, can be computed as

$$\mathbb{E}\left[\delta(\mathbf{s})\right] = \sum_{\mathbf{s}} \left(\delta(\mathbf{s})\pi(\mathbf{s})\right), \qquad (5)$$

where $\delta(s)$ is taken from (4) and vector $\boldsymbol{\pi}$ is according to (2).

The exact analysis derives the full statistics of AoI, i.e., it can compute higher-order moments beyond the average. As such, it can be obtain, for example, the AoI standard deviation, or the probability of peak AoI violation, which are important requirements in multimedia applications [7]. In these cases, the presented analysis is clearly convenient over simulation, as it assures precise evaluations even for extremely small values. Moreover, it can be integrated into larger approaches, such as optimizations of ARQ based on reinforcement learning [16].

IV. Numerical Results

To show some evaluations from the previous analysis, results from the closed-form (5) are displayed. Monte Carlo simulation results (not shown to avoid overcrowding the plots) were also computed and found to be in perfect agreement.

Fig. 2 reports the average AoI as a function of the probability of packet erasure. Four cases are shown, for two different round trip times (m=3 or m=7), as well as comparing i.i.d channel erasures with a correlated channel with average error burst length B=8. For all the curves, the trend is linearly increasing; this is interesting because it shows that SR ARQ error control is effective in keeping the AoI from exploding when the erasure probability ε increases. For stand-alone data content (without sequential decoding and SR ARQ) [30], increasing the error rate ε causes the average AoI to follow $(1-\varepsilon)^{-1}$, whose Taylor series for $\varepsilon<1$ is $1+\varepsilon+\varepsilon^2+o(\varepsilon^2)$, i.e., it causes a superlinear increase. It seems that the interdependencies among multiple packets, while overall increasing the average AoI, also regularize its growth as a function of the erasure probability. The average AoI increases with m with a superlinear growth; as m increases, it is slightly more likely to have multiple erasures that amplify the AoI surge, as they must be resolved to release all previous packets.

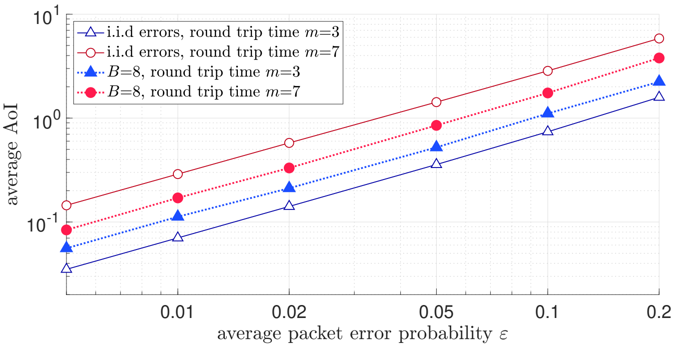

Fig. 2. Average AoI of SR ARQ, versus the error probability ε under different round trip times, for i.i.d erasures or average burst length of B=8.

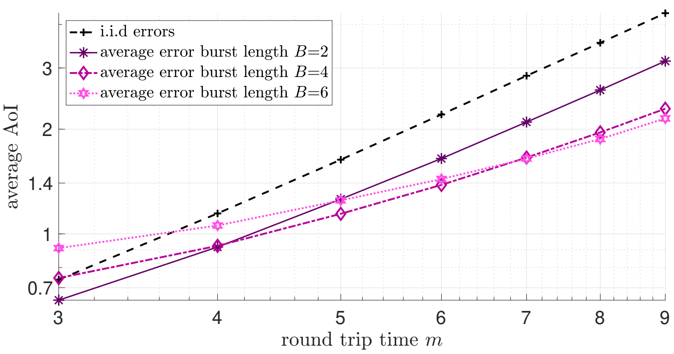

Fig. 3. Average AoI of SR ARQ, versus the round trip time m under different correlations, for error probability $\varepsilon=0.1$.

It is also interesting to see what happens in the case of channel error correlation [19]. Comparing i.i.d erasures with a correlated channel, one can see that the linear increasing trend of AoI is preserved, yet the curves get closer when errors are correlated; specifically, the average AoI is worse for m=3 but sees an improvement when m=7. The reason is that, for m=3, an error burst of 8 slots spans multiple round trip times, which increases the AoI. But for m=7, the error burst length is comparable with m, which decreases the average AoI, as will be explored in more detail in the following results.

The impact of the round trip time on AoI is explicitly considered in Fig. 3. Here, the average AoI is plotted versus m for error probability $\varepsilon=0.1$ and different levels of error correlation. This result confirms that the increasing linear trend of AoI has different slopes depending on the channel correlation, so that a channel with longer error bursts exhibits poorer performance at low round trip times, but it catches up as m increases and actually becomes better than an i.i.d channel. In other words, a mild error correlation can be helpful for AoI, which is in line with [28] that suggests to game the medium access to artificially introduce access correlation, i.e., stay silent until reaching an age threshold and then transmit

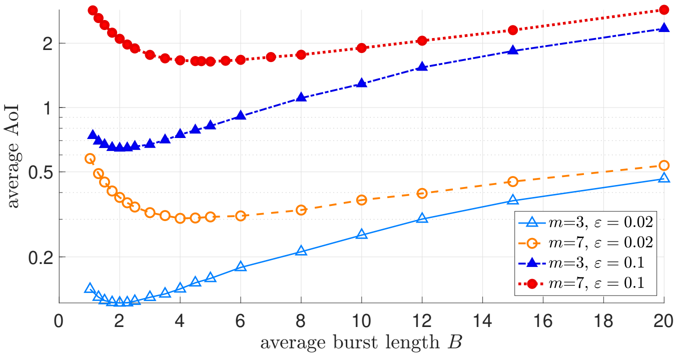

Fig. 4. Average AoI of SR ARQ, versus the average erasure burst length B for different choices of the error probability ε and the round trip time m.

in bursts. In our case, the reason for this improvement is that channel correlation for errors also implies that there are long sequences of slots without any erasure, which may result in the delivery of an entire window of packets. This is in line with results of ARQ pertaining to delay [13], [19].

Fig. 4 shows the dependence on the channel correlation through the average erasure burst length B. Different values of ε and m are chosen, to show that AoI attains a minimum for burst lengths intermediate between half and a full round trip time. Low AoI is achieved when the channel is good for many consecutive time slots; if the correlation is too acute, long streaks of correctly received packets are balanced by those undergoing multiple retransmissions (this is likely if B is much larger than the round trip time m), which makes AoI to soar. For strongly correlated channels, AoI tends to similar limits even for different values of the round trip time m. If the error bursts are long, the value of the round trip time becomes less important, and the AoI behavior is dominated by B.

V. Conclusions

A matrix-geometric analysis of SR ARQ was presented to derive the AoI statistics for sequential content that requires in-order delivery. Interesting conclusions were drawn about error correlation of limited extent being actually beneficial for AoI.

This serves to evaluate ARQ for what concerns timely status updates [17], and paves the road to deriving guidelines for deeper studies about various (hybrid) ARQ types [16], [18]. An interesting extension would be toward advanced ARQ schemes that take advantage of correlation to adapt the ARQ transmission [29]. One can also think of developing an *age-aware* ARQ scheme, in the same spirit of what done for transmission protocols at different layers [6], [28], which appears as an interesting yet challenging future work.

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