

# **“Dead or Alive, we can deny it”. A Differentially Private Approach to Survival Analysis.**

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# Medical Analysis and Privacy

## Introduction

- Clustering patients based on similar characteristics, such as diseases or treatments, to obtain insights for the research.
- Researchers calculate the **survival probability** of new patients belonging to the population computed.





# Medical Analysis and Privacy

## Introduction

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## Privacy Risks

*The more sensitive the analysis is, the higher the risk of leaking sensitive patients' information grows.*

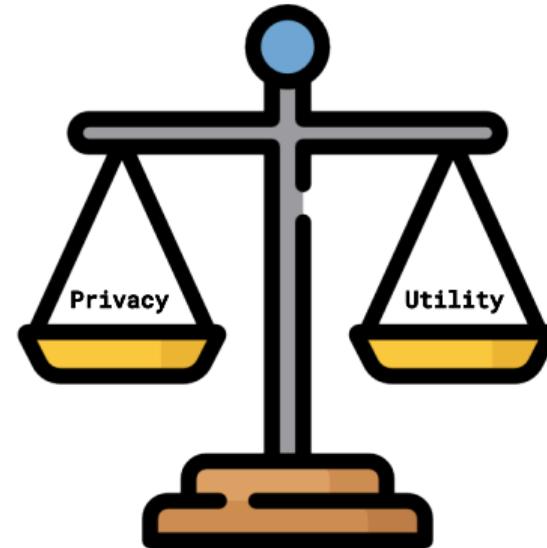




# Problem Statement

## Introduction

Can we apply  $\varepsilon$ -Differential Privacy and investigate the **Privacy vs. Utility trade-off** when performing Survival Analysis?





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# Survival Analysis

## Background & Related Works

Survival Analysis is a statistical technique used to analyze time-to-event or time-to-failure data when an event of interest has not yet occurred at time  $t$ .

Specifically, the Kaplan-Meier method [5] is used to evaluate the **survival trends** of the patients with common characteristics.

Kaplan-Meier Estimator:

$$\hat{S}(t) = \prod_{i:t_i \leq t} \left( \frac{n_i - d_i}{n_i} \right)$$

where:

$d_i$  is the event occurrence.

$n_i$  is the count of patients who have not experienced the event analysed at time  $t_i \leq t$ .



# $\varepsilon$ -Differential Privacy

## Background & Related Works

### Definition (Dwork et al. [2])

A randomized mechanism  $\mathcal{M}$ , i.e., an algorithm that takes an input and returns a noisy output, is  **$\varepsilon$ -Differentially Private** iff for any pair of neighbouring datasets  $D$  and  $D'$ , i.e., datasets that differ for at most one record, and a privacy budget  $\varepsilon \in \mathbb{R}^+$ , it holds:

$$\Pr [\mathcal{M}(D) \in S] \leq e^\varepsilon \cdot \Pr [\mathcal{M}(D') \in S] \quad \forall S \subset \text{Im}(\mathcal{M})$$

### Remark:

The lower the  $\varepsilon$ , the higher the privacy guarantees provided by the mechanism  $\mathcal{M}$ .



# Privacy Loss

## Background & Related Works

To verify that a mechanism  $\mathcal{M}$  is  $\varepsilon$ -Differentially Private, the **Privacy Loss** of the mechanism must be bounded by the privacy budget  $\varepsilon$  with probability 1 [3].

$$\mathcal{L}_{\mathcal{M}(D)||\mathcal{M}(D')}(O) = \log \left( \frac{\Pr [\mathcal{M}(D) = O]}{\Pr [\mathcal{M}(D') = O]} \right)$$

where:

$D$  and  $D'$  are neighbouring datasets.

$O$  is an output of the mechanism.



## Related Works

### Background & Related Works

The other proposed method (**LNT**E) to obfuscate Survival Analysis [4] was based on the Laplacian mechanism to change the number of subjects at risk and events in a dataset.

The algorithm iteratively **adjusts these counts** across time points to maintain updated risk and event information, returning a differentially private estimation of the survival rate.



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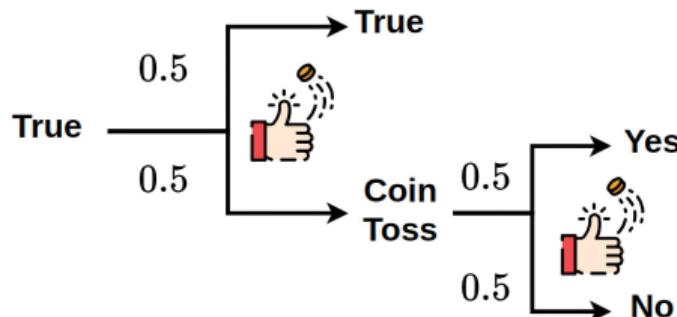
## Methodology

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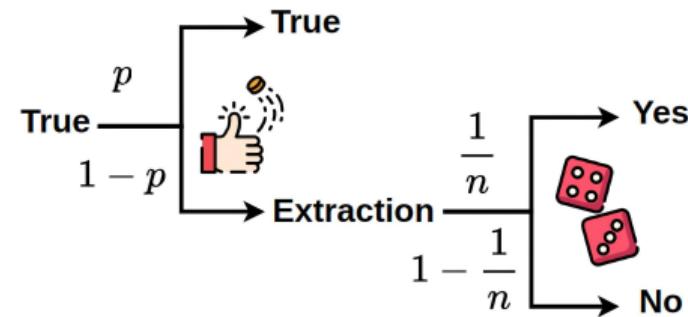


# Revised Randomized Response (RRR)

## Methodology



(a) Original Randomized Response mechanism.



(b) Revised Randomized Response mechanism.



# Privacy Properties

## Methodology

These equations provide the probability of obtaining as the output the real category  $C_i$ , considering the input category  $C_i$  or another category  $\bar{C}_i$ .

$$\Pr[Resp = C_i | True = C_i] = p + (1 - p) \frac{1}{n} = \frac{np + 1 - p}{n}$$

$$\Pr[Resp = C_i | True = \bar{C}_i] = (1 - p) \left(1 - \frac{1}{n}\right) = \frac{n - 1 - np + p}{n}$$



# Privacy Condition

## Methodology

We compute the Privacy Loss of the RRR mechanism:

$$\varepsilon(n, p) = \log \left( \frac{np + 1 - p}{n - 1 - np + p} \right)$$

Therefore, by **fixing** the number of categories  $n$ , the condition to satisfy the  $\varepsilon$ -Differential Privacy definition is:

$$\varepsilon > 0 \iff p \in \left( \frac{n-2}{2n-2}, 1 \right)$$



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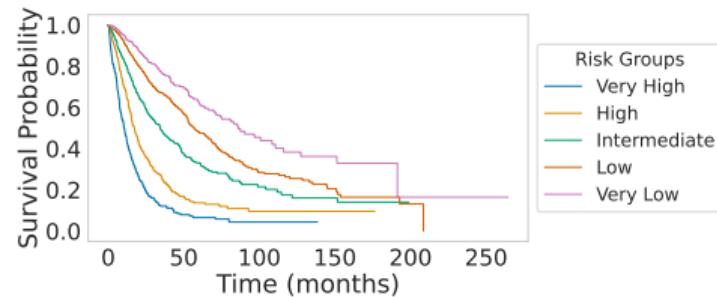
## Experimental Results

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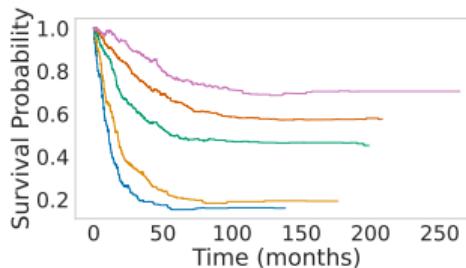


# Kaplan Meier Survival Curves: Original vs. LNTE

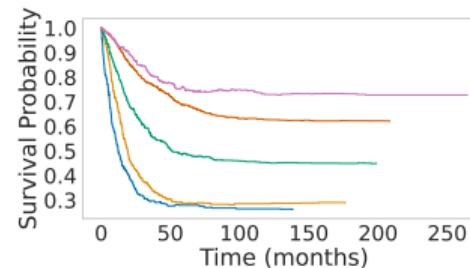
Experimental Results: IPSS-R Dataset [1]



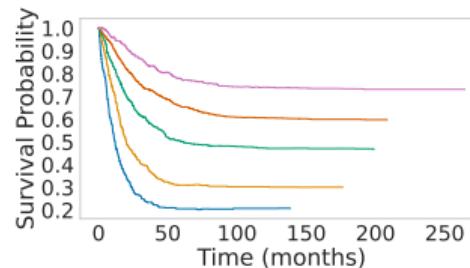
(c) IPSS-R



(d) LNTE,  $\varepsilon = 1$



(e) LNTE,  $\varepsilon = 2$

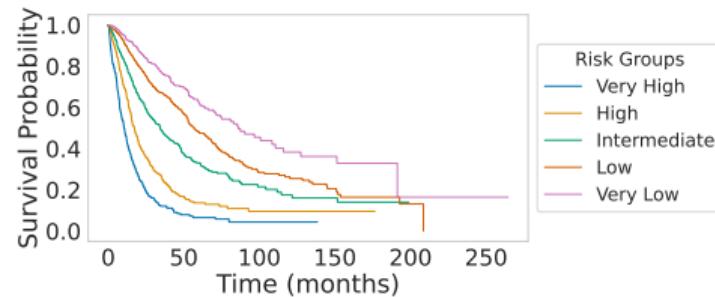


(f) LNTE,  $\varepsilon = 3$

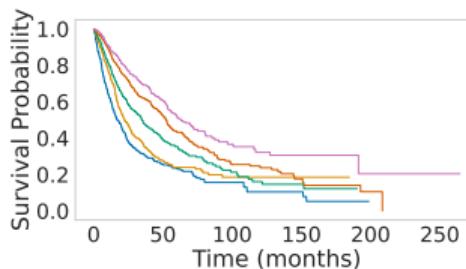


# Kaplan-Meier Survival Curves: Original vs. RRR

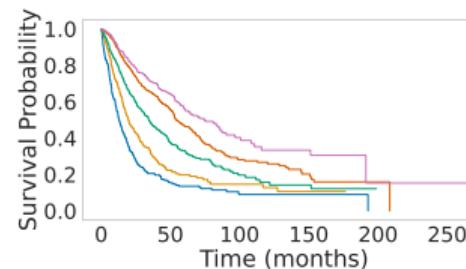
Experimental Results: IPSS-R Dataset [1]



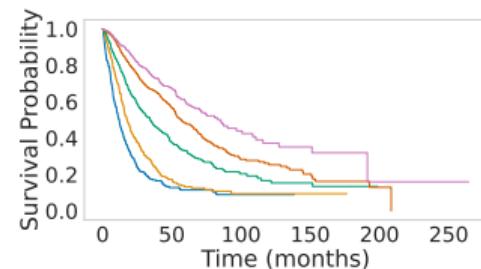
(g) IPSS-R



(h) RRR,  $\varepsilon = 1$



(i) RRR,  $\varepsilon = 2$



(j) RRR,  $\varepsilon = 3$



## Pairwise Log-Rank Test: Original vs. RRR

Experimental Results: Kidney Dataset [6]

Disease A	Disease B	Test Statistics		p-value		- $\log_2(p)$	
		Original	$\varepsilon = 3$	Original	$\varepsilon = 3$	Original	$\varepsilon = 3$
AN	GN	0.01	0.11	0.93	0.75	0.11	0.42
	Other	1.69	0.85	0.19	0.36	2.37	1.48
	PKD	1.09	0.79	0.30	0.37	1.75	1.42
GN	Other	0.99	0.63	0.32	0.43	1.64	1.23
	PKD	0.60	0.60	0.44	0.44	1.19	1.19
Other	PKD	0.26	0.39	0.61	0.53	0.71	0.91



# Median Survival times

## Experimental Results

Dataset	Mech.	Category	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 3$	No DP
Kidney	LNTE	AN	1.30 (1.13, -)	1.30 (1.00, -)	1.33 (1.00, -)	1.77
		GN	1.00 (0.50, -)	4.33 (0.73, -)	5.13 (0.87, -)	1.00
		PKD	18.73 (5.07, -)	5.07 (1.00, -)	5.07 (2.10, -)	2.60
		Other	3.97 (1.80, -)	5.90 (2.10, -)	8.17 (3.80, -)	4.70
	RRR	AN	2.20 (1.27, 6.53)	1.43 (0.90, 3.20)	1.77 (0.90, 3.20)	1.77
		GN	0.93 (0.40, 4.33)	1.27 (0.50, 5.20)	1.30 (0.50, 5.20)	1.00
		PKD	2.60 (0.50, 9.73)	5.07 (0.87, 17.03)	4.40 (1.00, 5.07)	2.60
		Other	5.07 (0.80, 14.9)	3.97 (0.80, 9.73)	4.70 (0.93, 8.17)	4.70



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# Contributions & Future Work

## Conclusion

Our findings suggest that the RRR mechanism:

- Results in a more effective privacy and utility balance.
- Maintains the distribution properties of real results.
- Offers a new comparison method for future investigations.

As Future Directions:

- Investigate further the privacy guarantees offered by the Differential Privacy mechanism.
- Propose new privacy strategies for bridging the gap between Survival Analysis and Differential Privacy, e.g., Linear and Cox Regression with Differential Privacy.

# “Dead or Alive, we can deny it”. A Differentially Private Approach to Survival Analysis.

*Thank you for listening!*  
*Any questions?*



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